The tables gives expressions for $\mathrm{VaR}_p(X)$ and $\mathrm{ES}_p(X)$ when $X$ is an absolutely continuous random variable specified by the stated pdf and cdf.

| | pdf | cdf | $\mathrm{VaR}_p(X)$ | $\mathrm{ES}_p(X)$ |
|---|---|---|---|---|
| Exponential | $\lambda \exp(-\lambda x)$, $x > 0$ | $1-\exp(-\lambda x)$ | $-\frac{1}{\lambda}\log(1-p)$ | $-\frac{1}{p\lambda}\left\{\log(1-p)p - p - \log(1-p)\right\}$ |
| Kumaraswamy exponential | $ab\lambda\exp(-\lambda x) \cdot [1-\exp(-\lambda x)]^{a-1} \cdot \{1-[1-\exp(-\lambda x)]^a\}^{b-1}$, $x>0$ | $1-\left\{1-\left[1-\exp(-\lambda x)\right]^a\right\}^b$ | $-\frac{1}{\lambda}\log\left\{1-\left[1-(1-p)^{1/b}\right]^{1/a}\right\}$ | $-\frac{1}{p\lambda}\int_0^p \log\left\{1-\left[1-(1-v)^{1/b}\right]^{1/a}\right\}dv$ |
| Exponentiated exponential | $\alpha\lambda\exp(-\lambda x) \cdot [1-\exp(-\lambda x)]^{\alpha-1}$, $x>0$ | $[1-\exp(-\lambda x)]^{\alpha}$ | $-\frac{1}{\lambda}\log\left(1-p^{1/\alpha}\right)$ | $-\frac{1}{p\lambda}\int_0^p \log\left(1-v^{1/\alpha}\right)dv$ |
| Inverse exponentiated exponential | $\alpha\lambda x^{-2}\exp\left(-\frac{\lambda}{x}\right) \cdot \left[1-\exp\left(-\frac{\lambda}{x}\right)\right]^{\alpha-1}$, $x>0$ | $1-\left[1-\exp\left(-\frac{\lambda}{x}\right)\right]^{\alpha}$ | $\lambda\left\{-\log\left[1-(1-p)^{1/\alpha}\right]\right\}^{-1}$ | $\frac{\lambda}{p}\int_0^p\left\{-\log\left[1-(1-v)^{1/\alpha}\right]\right\}^{-1}dv$ |
| Beta exponential | $\frac{\lambda\exp(-b\lambda x)}{B(a,b)} \cdot [1-\exp(-\lambda x)]^{a-1}$, $x>0$ | $I_{1-\exp(-\lambda x)}(a,b)$ | $-\frac{1}{\lambda}\log\left[1-I_p^{-1}(a,b)\right]$ | $-\frac{1}{p\lambda}\int_0^p\log\left[1-I_v^{-1}(a,b)\right]dv$ |
| Logistic exponential | $\frac{\alpha\lambda\exp(\lambda x)\left[\exp(\lambda x)-1\right]^{\alpha-1}}{\left\{1+\left[\exp(\lambda x)-1\right]^{\alpha}\right\}^2}$, $x>0$ | $\frac{[\exp(\lambda x)-1]^{\alpha}}{1+[\exp(\lambda x)-1]^{\alpha}}$ | $\frac{1}{\lambda}\log\left[1+\left(\frac{p}{1-p}\right)^{1/\alpha}\right]$ | $\frac{1}{p\lambda}\int_0^p\log\left[1+\left(\frac{v}{1-v}\right)^{1/\alpha}\right]dv$ |
| Exponential extension | $\alpha\lambda(1+\lambda x)^{\alpha-1} \cdot \exp\left[1-(1+\lambda x)^{\alpha}\right]$, $x>0$ | $1-\exp\left[1-(1+\lambda x)^{\alpha}\right]$ | $\frac{[1-\log(1-p)]^{1/\alpha}-1}{\lambda}$ | $-\frac{1}{\lambda}+\frac{1}{\lambda p}\int_0^p[1-\log(1-v)]^{1/\alpha}dv$ |
| Marshall-Olkin exponential | $\frac{\lambda\exp(\lambda x)}{[\exp(\lambda x)-1+\alpha]^2}$, $x>0$ | $\frac{\exp(\lambda x)-2+\alpha}{\exp(\lambda x)-1+\alpha}$ | $\frac{1}{\lambda}\log\frac{2-\alpha-(1-\alpha)p}{1-p}$ | $\frac{1}{\lambda}\log[2-\alpha-(1-\alpha)p] - \frac{2-\alpha}{\lambda(1-\alpha)p} \cdot \log\frac{2-\alpha-(1-\alpha)p}{2-\alpha} + \frac{1-p}{\lambda p}\log(1-p)$ |
| Perks | $\frac{\alpha\exp(\beta x)[1+\alpha]}{[1+\alpha\exp(\beta x)]^2}$, $x>0$ | $1-\frac{1+\alpha}{1+\alpha\exp(\beta x)}$ | $\frac{1}{\beta}\log\frac{\alpha+p}{\alpha(1-p)}$ | $-\left(1+\frac{\alpha}{p}\right)\frac{\log\alpha}{\beta} + \frac{(\alpha+p)\log(\alpha+p)}{\beta p} + \frac{(1-p)\log(1-p)}{\beta p}$ |
| Beard | $\frac{\alpha\exp(\beta x)[1+\alpha\rho]^{\rho^{-1/\beta}}}{[1+\alpha\rho\exp(\beta x)]^{1+\rho^{-1/\beta}}}$, $x>0$ | $1-\frac{[1+\alpha\rho]^{\rho^{-1/\beta}}}{[1+\alpha\rho\exp(\beta x)]^{\rho^{-1/\beta}}}$ | $\frac{1}{\beta}\log\left[\frac{1+\alpha\rho}{\alpha\rho(1-p)^{\rho^{1/\beta}}}-\frac{1}{\alpha\rho}\right]$ | $\frac{1}{p\beta}\int_0^p\log\left[-\frac{1}{\alpha\rho}+\frac{1+\alpha\rho}{\alpha\rho(1-v)^{\rho^{1/\beta}}}\right]dv$ |
| Gompertz | $b\eta\exp(bx) \cdot \exp\left[\eta-\eta\exp(bx)\right]$, $x>0$ | $1-\exp\left[\eta-\eta\exp(bx)\right]$ | $\frac{1}{b}\log\left[1-\frac{1}{\eta}\log(1-p)\right]$ | $\frac{1}{pb}\int_0^p\log\left[1-\frac{1}{\eta} \cdot \log(1-v)\right]dv$ |
| Beta Gompertz | $\frac{b\eta\exp(bx)}{B(c,d)} \cdot \exp(d\eta) \cdot \exp\left[-d\eta\exp(bx)\right] \cdot \left\{1-\exp\left[\eta-\eta\exp(bx)\right]\right\}^{c-1}$, $x>0$ | $I_{1-\exp[\eta-\eta\exp(bx)]}(c,d)$ | $\frac{1}{b}\log\left\{1-\frac{1}{\eta} \cdot \log\left[1-I_p^{-1}(c,d)\right]\right\}$ | $\frac{1}{pb}\int_0^p\log\left\{1-\frac{1}{\eta}\log\left[1-I_v^{-1}(c,d)\right]\right\}dv$ |
| Linear failure rate | $(a+bx) \cdot \exp\left(-ax-bx^2/2\right)$, $x>0$ | $1-\exp\left(-ax-bx^2/2\right)$ | $\frac{-a+\sqrt{a^2-2b\log(1-p)}}{b}$ | $-\frac{a}{b}+\frac{1}{bp}\int_0^p\sqrt{a^2-2b\log(1-v)}dv$ |
| Pareto | $cK^cx^{-c-1}$, $x \geq K$ | $1-\left(\frac{K}{x}\right)^c$ | $K(1-p)^{-1/c}$ | $\frac{Kc}{p(1-c)}(1-p)^{1-1/c} - \frac{Kc}{p(1-c)}$ |

| | | | | |
|---|---|---|---|---|
| Kumaraswamy Pareto | $abcK^c x^{-c-1} \cdot \left[1-\left(\frac{K}{x}\right)^c\right]^{a-1} \cdot \left\{1-\left[1-\left(\frac{K}{x}\right)^c\right]^a\right\}^{b-1}$, $x \geq K$ | $1-\left\{1-\left[1-\left(\frac{K}{x}\right)^c\right]^a\right\}^b$ | $K\left\{1-\left[1-(1-p)^{1/b}\right]^{1/a}\right\}^{-1/c}$ | $\frac{K}{p}\int_0^p \left\{1-\left[1-(1-v)^{1/b}\right]^{1/a}\right\}^{-1/c} dv$ |
| $F$ | $\frac{1}{B\left(\frac{d_1}{2},\frac{d_2}{2}\right)} \cdot \left(\frac{d_1}{d_2}\right)^{\frac{d_1}{2}} \cdot x^{\frac{d_1}{2}-1} \cdot \left(1+\frac{d_1}{d_2}x\right)^{-\frac{d_1+d_2}{2}}$, $x>0$ | $I_{\frac{d_1 x}{d_1 x+d_2}}\left(\frac{d_1}{2},\frac{d_2}{2}\right)$ | $\frac{d_2}{d_1}\frac{I_p^{-1}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}{1-I_p^{-1}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}$ | $\frac{d_2}{d_1 p}\int_0^p \frac{I_v^{-1}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}{1-I_v^{-1}\left(\frac{d_1}{2},\frac{d_2}{2}\right)} dv$ |
| Generalized Pareto | $\frac{1}{k}\left(1-\frac{cx}{k}\right)^{1/c-1}$, $x<k/c$ if $c>0$, $x>k/c$ if $c<0$, $x>0$ if $c=0$ | $1-\left(1-\frac{cx}{k}\right)^{1/c}$ | $\frac{k}{c}\left[1-(1-p)^c\right]$ | $\frac{k}{c}+\frac{k(1-p)^{c+1}}{pc(c+1)}-\frac{k}{pc(c+1)}$ |
| Beta Pareto | $\frac{aK^{ad}x^{-ad-1}}{B(c,d)} \cdot \left[1-\left(\frac{K}{x}\right)^a\right]^{c-1}$, $x \geq K$ | $I_{1-\left(\frac{K}{x}\right)^a}(c,d)$ | $K\left[1-I_p^{-1}(c,d)\right]^{-1/a}$ | $\frac{K}{p}\int_0^p \left[1-I_v^{-1}(c,d)\right]^{-1/a} dv$ |
| Pareto positive stable | $\frac{\nu\lambda}{x}\left[\log\left(\frac{x}{\sigma}\right)\right]^{\nu-1} \cdot \exp\left\{-\lambda\left[\log\left(\frac{x}{\sigma}\right)\right]^\nu\right\}$, $x>0$ | $1-\exp\left\{-\lambda\left[\log\left(\frac{x}{\sigma}\right)\right]^\nu\right\}$ | $\sigma\exp\left\{\left[-\frac{1}{\lambda}\log(1-p)\right]^{1/\nu}\right\}$ | $\frac{\sigma}{p}\int_0^p \exp\left\{\left[-\frac{1}{\lambda}\log(1-v)\right]^{1/\nu}\right\}dv$ |
| Gamma | $\frac{\beta^\alpha x^{\alpha-1}\exp(-\beta x)}{\Gamma(\alpha)}$, $x>0$ | $\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)}$ | $\frac{1}{\beta}Q^{-1}(\alpha,1-p)$ | $\frac{1}{\beta p}\int_0^p Q^{-1}(\alpha,1-v)dv$ |
| Kumaraswamy gamma | $ab\beta^\alpha x^{\alpha-1}\exp(-\beta x) \cdot \frac{\gamma^{a-1}(\alpha,\beta x)}{\Gamma^a(\alpha)} \cdot \left[1-\frac{\gamma^a(\alpha,\beta x)}{\Gamma^a(\alpha)}\right]^{b-1}$, $x>0$ | $1-\left[1-\frac{\gamma^a(\alpha,\beta x)}{\Gamma^a(\alpha)}\right]^b$ | $\frac{1}{\beta}Q^{-1}\left(\alpha,1-\left[1-(1-p)^{1/b}\right]^{1/a}\right)$ | $\frac{1}{\beta p}\int_0^p Q^{-1}\left(\alpha,1-\left[1-(1-v)^{1/b}\right]^{1/a}\right)dv$ |
| Nakagami | $\frac{2m^m}{\Gamma(m)a^m}x^{2m-1} \cdot \exp\left(-\frac{mx^2}{a}\right)$, $x>0$ | $1-Q\left(m,\frac{mx^2}{a}\right)$ | $\sqrt{\frac{a}{m}}\sqrt{Q^{-1}(m,1-p)}$ | $\frac{\sqrt{a}}{p\sqrt{m}}\int_0^p \sqrt{Q^{-1}(m,1-v)}dv$ |
| Reflected gamma | $\frac{1}{2\phi\Gamma(\alpha)}\left\lvert\frac{x-\theta}{\phi}\right\rvert^{\alpha-1} \cdot \exp\left\{-\left\lvert\frac{x-\theta}{\phi}\right\rvert\right\}$, $-\infty<x<\infty$ | $\begin{cases}\frac{1}{2}Q\left(\alpha,\frac{\theta-x}{\phi}\right), & \text{if } x \leq \theta, \\ 1-\frac{1}{2}Q\left(\alpha,\frac{x-\theta}{\phi}\right), & \text{if } x>\theta\end{cases}$ | $\begin{cases}\theta-\phi Q^{-1}(\alpha,2p), & \text{if } p \leq 1/2, \\ \theta+\phi Q^{-1}(\alpha,2(1-p)), & \text{if } p>1/2\end{cases}$ | $\begin{cases}\theta-\frac{\phi}{p}\int_0^p Q^{-1}(\alpha,2v)dv, & \text{if } p \leq 1/2, \\ \theta-\frac{\phi}{p}\int_0^{1/2} Q^{-1}(\alpha,2v)dv+\frac{\phi}{p}\int_{1/2}^p Q^{-1}(\alpha,2(1-v))dv, & \text{if } p>1/2\end{cases}$ |
| Compound Laplace gamma | $\frac{\alpha\beta}{2}\left\{1+\beta\lvert x-\theta\rvert\right\}^{-(\alpha+1)}$, $-\infty<x<\infty$ | $\begin{cases}\frac{1}{2}\left\{1+\beta\lvert x-\theta\rvert\right\}^{-\alpha}, & \text{if } x \leq \theta, \\ 1-\frac{1}{2}\left\{1+\beta\lvert x-\theta\rvert\right\}^{-\alpha}, & \text{if } x>\theta\end{cases}$ | $\begin{cases}\theta-\frac{1}{\beta}-\frac{(2p)^{-1/\alpha}}{\beta}, & \text{if } p \leq 1/2, \\ \theta-\frac{1}{\beta}+\frac{(2(1-p))^{-1/\alpha}}{\beta}, & \text{if } p>1/2\end{cases}$ | $\begin{cases}\theta-\frac{1}{\beta}-\frac{(2p)^{-1/\alpha}}{\beta(1-1/\alpha)}, & \text{if } p \leq 1/2, \\ \theta-\frac{1}{\beta}-\frac{[2(1-p)]^{1-1/\alpha}}{2p\beta(1-1/\alpha)}, & \text{if } p>1/2\end{cases}$ |
| Log gamma | $\frac{\alpha^r x^{\alpha-1}(-\log x)^{r-1}}{\Gamma(r)}$, $x>0$ | $Q(r,-\alpha\log x)$ | $\exp\left[-\frac{1}{\alpha}Q^{-1}(r,p)\right]$ | $\frac{1}{p}\int_0^p \exp\left[-\frac{1}{\alpha}Q^{-1}(r,v)\right]dv$ |

| | | | | |
|---|---|---|---|---|
| Inverse gamma | $\frac{\beta^{\alpha}\exp(-\beta/x)}{x^{\alpha+1}\Gamma(\alpha)}$, $x>0$ | $Q(\alpha,\beta/x)$ | $\beta\left[Q^{-1}(\alpha,p)\right]^{-1}$ | $\frac{\beta}{p}\int_0^p \left[Q^{-1}(\alpha,v)\right]^{-1}dv$ |
| Stacy | $\frac{cx^{c\gamma-1}\exp\left[-(x/\theta)^c\right]}{\theta^{c\gamma}\Gamma(\gamma)}$, $x>0$ | $1-Q\left(\gamma,\left(\frac{x}{\theta}\right)^c\right)$ | $\theta\left[Q^{-1}(\gamma,1-p)\right]^{1/c}$ | $\frac{\theta}{p}\int_0^p \left[Q^{-1}(\gamma,1-v)\right]^{1/c}dv$ |
| Lindley | $\frac{\lambda^2}{1+\lambda}\cdot(1+x)\cdot\exp(-\lambda x)$, $x>0$ | $1-\frac{1+\lambda+\lambda x}{1+\lambda}\cdot\exp(-\lambda x)$ | $-1-\frac{1}{\lambda}-\frac{1}{\lambda}\cdot W\Big(-(1+\lambda)\cdot(1-p)\cdot\exp(-1-\lambda)\Big)$ | $-1-\frac{1}{\lambda}-\frac{1}{p\lambda}\cdot\int_0^p W\Big(-(1+\lambda)\cdot(1-v)\cdot\exp(-1-\lambda)\Big)dv$ |
| Generalized Lindley | $\frac{\alpha\lambda^2}{1+\lambda}\cdot(1+x)\cdot\left[1-\frac{1+\lambda+\lambda x}{1+\lambda}\cdot\exp(-\lambda x)\right]^{\alpha-1}\cdot\exp(-\lambda x)$, $x>0$ | $\left[1-\frac{1+\lambda+\lambda x}{1+\lambda}\cdot\exp(-\lambda x)\right]^{\alpha}$ | $-1-\frac{1}{\lambda}-\frac{1}{\lambda}\cdot W\Big(-(1+\lambda)\cdot\left(1-p^{1/\alpha}\right)\cdot\exp(-1-\lambda)\Big)$ | $-1-\frac{1}{\lambda}-\frac{1}{p\lambda}\cdot\int_0^p W\Big(-(1+\lambda)\cdot\left(1-v^{1/\alpha}\right)\cdot\exp(-1-\lambda)\Big)dv$ |
| Beta | $\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$, $0\le x\le 1$ | $I_x(a,b)$ | $I_p^{-1}(a,b)$ | $\frac{1}{p}\int_0^p I_v^{-1}(a,b)dv$ |
| Uniform | $\frac{1}{b-a}$, $a\le x\le b$ | $\frac{x-a}{b-a}$ | $a+p(b-a)$ | $a+\frac{p}{2}(b-a)$ |
| Generalized uniform | $hkc(x-a)^{c-1}\cdot[1-k(x-a)^c]^{h-1}$, $a\le x\le a+k^{-1/c}$ | $1-[1-k(x-a)^c]^h$ | $a+k^{-1/c}\left[1-(1-p)^{1/h}\right]^{1/c}$ | $a+\frac{k^{-1/c}}{p}\cdot\int_0^p \left[1-(1-v)^{1/h}\right]^{1/c}dv$ |
| Power function I | $ax^{a-1}$, $0\le x\le 1$ | $x^a$ | $p^{1/a}$ | $\frac{p^{1/a}}{1/a+1}$ |
| Power function II | $b(1-x)^{b-1}$, $0\le x\le 1$ | $1-(1-x)^b$ | $1-(1-p)^{1/b}$ | $1+\frac{b\left[(1-p)^{1/b+1}-1\right]}{p(b+1)}$ |
| Log beta | $\frac{(\log d-\log c)^{1-a-b}}{xB(a,b)}\cdot(\log x-\log c)^{a-1}\cdot(\log d-\log x)^{b-1}$, $c\le x\le d$ | $I_{\frac{\log x-\log c}{\log d-\log c}}(a,b)$ | $c\left(\frac{d}{c}\right)^{I_p^{-1}(a,b)}$ | $\frac{c}{p}\int_0^p \left(\frac{d}{c}\right)^{I_v^{-1}(a,b)}dv$ |
| Complementary beta | $B(a,b)\cdot\left\{I_x^{-1}(a,b)\right\}^{1-a}\cdot\left\{1-I_x^{-1}(a,b)\right\}^{1-b}$, $0\le x\le 1$ | $I_x^{-1}(a,b)$ | $I_p(a,b)$ | $\frac{1}{p}\int_0^p I_v(a,b)dv$ |
| Libby-Novick beta | $\frac{\lambda^a x^{a-1}(1-x)^{b-1}}{B(a,b)[1-(1-\lambda)x]^{a+b}}$, $0\le x\le 1$ | $I_{\frac{\lambda x}{1+(\lambda-1)x}}(a,b)$ | $\frac{I_p^{-1}(a,b)}{\lambda-(\lambda-1)I_p^{-1}(a,b)}$ | $\frac{1}{p}\int_0^p \frac{I_v^{-1}(a,b)}{\lambda-(\lambda-1)I_v^{-1}(a,b)}dv$ |
| McDonald-Richards beta | $\frac{x^{ar-1}(bq^r-x^r)^{b-1}}{(bq^r)^{a+b-1}B(a,b)}$, $0\le x\le b^{1/r}q$ | $I_{\frac{x^r}{bq^r}}(a,b)$ | $b^{1/r}q\left[I_p^{-1}(a,b)\right]^{1/r}$ | $\frac{b^{1/r}q}{p}\int_0^p \left[I_v^{-1}(a,b)\right]^{1/r}dv$ |
| Generalized beta | $\frac{(x-c)^{a-1}(d-x)^{b-1}}{B(a,b)(d-c)^{a+b-1}}$, $c\le x\le d$ | $I_{\frac{x-c}{d-c}}(a,b)$ | $c+(d-c)I_p^{-1}(a,b)$ | $c+\frac{d-c}{p}\int_0^p I_v^{-1}(a,b)dv$ |
| Arcsine | $\frac{1}{\pi\sqrt{(x-a)(b-x)}}$, $a\le x\le b$ | $\frac{2}{\pi}\arcsin\left(\sqrt{\frac{x-a}{b-a}}\right)$ | $a+(b-a)\sin^2\left(\frac{\pi p}{2}\right)$ | $a+\frac{b-a}{p}\int_0^p \sin^2\left(\frac{\pi v}{2}\right)dv$ |

| | | | | |
|---|---|---|---|---|
| Triangular | $\begin{cases} 0, & \text{if } x < a, \\ \frac{2(x-a)}{(b-a)(c-a)}, & \text{if } a \le x \le c, \\ \frac{2(b-x)}{(b-a)(b-c)}, & \text{if } c < x \le b, \\ 0, & \text{if } b < x \end{cases}$ | $\begin{cases} 0, & \text{if } x < a, \\ \frac{(x-a)^2}{(b-a)(c-a)}, & \text{if } a \le x \le c, \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, & \text{if } c < x \le b, \\ 1, & \text{if } b < x \end{cases}$ | $\begin{cases} a + \sqrt{p(b-a)(c-a)}, & \text{if } 0 < p < \frac{c-a}{b-a}, \\ b - \sqrt{(1-p)(b-a)(b-c)}, & \text{if } \frac{c-a}{b-a} \le p < 1 \end{cases}$ | $\begin{cases} a + \frac{2}{3} \cdot \sqrt{p(b-a)(c-a)}, & \text{if } 0 < p < \frac{c-a}{b-a}, \\ b + \frac{a-c}{p} + \frac{2(2c-a-b)}{3p} + 2\sqrt{(b-a)(b-c)} \cdot \frac{(1-p)^{3/2}}{3p}, & \text{if } \frac{c-a}{b-a} \le p < 1 \end{cases}$ |
| Generalized beta II | $\frac{cx^{ac-1}(1-x^c)^{b-1}}{B(a,b)}$, $0 \le x \le 1$ | $I_{x^c}(a,b)$ | $\left[I_p^{-1}(a,b)\right]^{1/c}$ | $\frac{1}{p}\int_0^p \left[I_v^{-1}(a,b)\right]^{1/c} dv$ |
| Inverse beta | $\frac{x^{a-1}}{B(a,b)(1+x)^{a+b}}$, $x > 0$ | $I_{\frac{x}{1+x}}(a,b)$ | $\frac{I_p^{-1}(a,b)}{1-I_p^{-1}(a,b)}$ | $\frac{1}{p}\int_0^p \frac{I_v^{-1}(a,b)}{1-I_v^{-1}(a,b)} dv$ |
| Generalized inverse beta | $\frac{ax^{ac-1}}{B(c,d)(1+x^a)^{c+d}}$, $x > 0$ | $I_{\frac{x^a}{1+x^a}}(c,d)$ | $\left[\frac{I_p^{-1}(c,d)}{1-I_p^{-1}(c,d)}\right]^{1/a}$ | $\frac{1}{p}\int_0^p \left[\frac{I_v^{-1}(c,d)}{1-I_v^{-1}(c,d)}\right]^{1/a} dv$ |
| Two sided power | $\begin{cases} a\left(\frac{x}{\theta}\right)^{a-1}, & \text{if } 0 < x \le \theta, \\ a\left(\frac{1-x}{1-\theta}\right)^{a-1}, & \text{if } \theta < x < 1 \end{cases}$ | $\begin{cases} \theta\left(\frac{x}{\theta}\right)^{a}, & \text{if } 0 < x \le \theta, \\ 1-(1-\theta)\left(\frac{1-x}{1-\theta}\right)^{a}, & \text{if } \theta < x < 1 \end{cases}$ | $\begin{cases} \theta\left(\frac{p}{\theta}\right)^{1/a}, & \text{if } 0 < p \le \theta, \\ 1-(1-\theta)\left(\frac{1-p}{1-\theta}\right)^{1/a}, & \text{if } \theta < p < 1 \end{cases}$ | $\begin{cases} \frac{a\theta}{a+1}\left(\frac{p}{\theta}\right)^{1/a}, & \text{if } 0 < p \le \theta, \\ 1 - \frac{\theta}{p} + \frac{a(2\theta-1)}{(a+1)p} + \frac{a(1-\theta)^2}{(a+1)p} \cdot \left(\frac{1-p}{1-\theta}\right)^{1+1/a}, & \text{if } \theta < p < 1 \end{cases}$ |
| Kumaraswamy | $abx^{a-1}\left(1-x^a\right)^{b-1}$, $0 \le x \le 1$ | $1-(1-x^a)^b$ | $\left[1-(1-p)^{1/b}\right]^{1/a}$ | $\frac{1}{p}\int_0^p \left[1 - (1-v)^{1/b}\right]^{1/a} dv$ |
| Normal | $\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$, $-\infty < x < \infty$ | $\Phi\left(\frac{x-\mu}{\sigma}\right)$ | $\mu + \sigma\Phi^{-1}(p)$ | $\mu + \frac{\sigma}{p}\int_0^p \Phi^{-1}(v)dv$ |
| Kumaraswamy normal | $\frac{ab}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right) \cdot \Phi^{a-1}\left(\frac{x-\mu}{\sigma}\right) \cdot \left[1-\Phi^a\left(\frac{x-\mu}{\sigma}\right)\right]^{b-1}$, $-\infty < x < \infty$ | $1 - \left[1 - \Phi^a\left(\frac{x-\mu}{\sigma}\right)\right]^b$ | $\mu + \sigma\Phi^{-1}\left(\left[1 - (1-p)^{1/b}\right]^{1/a}\right)$ | $\mu + \frac{\sigma}{p}\int_0^p \Phi^{-1}\left(\left[1-(1-v)^{1/b}\right]^{1/a}\right)dv$ |
| Exponential power | $\frac{1}{2a^{1/a}\sigma\Gamma(1+1/a)} \cdot \exp\left\{-\frac{\lvert x-\mu\rvert^a}{a\sigma^a}\right\}$, $-\infty < x < \infty$ | $\begin{cases} \frac{1}{2}Q\left(\frac{1}{a}, \frac{(\mu-x)^a}{a\sigma^a}\right), & \text{if } x \le \mu, \\ 1 - \frac{1}{2}Q\left(\frac{1}{a}, \frac{(x-\mu)^a}{a\sigma^a}\right), & \text{if } x > \mu \end{cases}$ | $\begin{cases} \mu - a^{1/a}\sigma \cdot \left[Q^{-1}\left(\frac{1}{a}, 2p\right)\right]^{1/a}, & \text{if } p \le 1/2, \\ \mu + a^{1/a}\sigma \cdot \left[Q^{-1}\left(\frac{1}{a}, 2(1-p)\right)\right]^{1/a}, & \text{if } p > 1/2 \end{cases}$ | $\begin{cases} \mu - \frac{a^{1/a}\sigma}{p} \cdot \int_0^p \left[Q^{-1}\left(\frac{1}{a}, 2v\right)\right]^{1/a} dv, & \text{if } p \le 1/2, \\ \mu - \frac{a^{1/a}\sigma}{p} \cdot \int_0^{1/2} \left[Q^{-1}\left(\frac{1}{a}, 2v\right)\right]^{1/a} dv + \frac{a^{1/a}\sigma}{p} \cdot \int_{1/2}^p \left[Q^{-1}\left(\frac{1}{a}, 2(1-v)\right)\right]^{1/a} dv, & \text{if } p > 1/2 \end{cases}$ |

| | | | | |
|---|---|---|---|---|
| Skewed exponential power | $\begin{cases} K(q) \cdot \exp\left[-\frac{1}{q}\left\lvert\frac{x}{2\alpha}\right\rvert^{q}\right], & \text{if } x \le 0, \\ K(q) \cdot \exp\left[-\frac{1}{q}\left\lvert\frac{x}{2-2\alpha}\right\rvert^{q}\right], & \text{if } x > 0, \\ \text{where } K(q) = \frac{1}{2q^{1/q}\Gamma(1+1/q)} \end{cases}$ | $\begin{cases} \alpha Q\left(\frac{1}{q}\left(\frac{\lvert x\rvert}{2\alpha}\right)^{q}, \frac{1}{q}\right), & \text{if } x \le 0, \\ 1-(1-\alpha) \cdot Q\left(\frac{1}{q}\left(\frac{\lvert x\rvert}{2-2\alpha}\right)^{q}, \frac{1}{q}\right), & \text{if } x > 0 \end{cases}$ | $\begin{cases} -2\alpha\left[qQ^{-1}\left(\frac{p}{\alpha}, \frac{1}{q}\right)\right]^{\frac{1}{q}}, & \text{if } p \le \alpha, \\ 2(1-\alpha) \cdot \left[qQ^{-1}\left(\frac{1-p}{1-\alpha}, \frac{1}{q}\right)\right]^{\frac{1}{q}}, & \text{if } p > \alpha \end{cases}$ | $\begin{cases} -\frac{2\alpha}{p}\int_0^p \left[qQ^{-1}\left(\frac{v}{\alpha}, \frac{1}{q}\right)\right]^{\frac{1}{q}} dv, & \text{if } p \le \alpha, \\ -\frac{2\alpha}{p}\int_0^\alpha \left[qQ^{-1}\left(\frac{v}{\alpha}, \frac{1}{q}\right)\right]^{\frac{1}{q}} dv + \frac{2(1-\alpha)}{p}\int_\alpha^p \left[qQ^{-1}\left(\frac{1-v}{1-\alpha}, \frac{1}{q}\right)\right]^{\frac{1}{q}} dv, & \text{if } p > \alpha \end{cases}$ |
| Asymmetric exponential power | $\begin{cases} \frac{\alpha}{\alpha^*}K(q_1) \cdot \exp\left[-\frac{1}{q_1}\left\lvert\frac{x}{2\alpha^*}\right\rvert^{q_1}\right], & \text{if } x \le 0, \\ \frac{1-\alpha}{1-\alpha^*}K(q_2) \cdot \exp\left[-\frac{1}{q_2}\left\lvert\frac{x}{2-2\alpha^*}\right\rvert^{q_2}\right], & \text{if } x > 0, \\ \text{where } K(q) = \frac{1}{2q^{1/q}\Gamma(1+1/q)}, \\ \alpha^* = \frac{\alpha K(q_1)}{\alpha K(q_1)+(1-\alpha)K(q_2)} \end{cases}$ | $\begin{cases} \alpha Q\left(\frac{1}{q_1}\left(\frac{\lvert x\rvert}{2\alpha^*}\right)^{q_1}, \frac{1}{q_1}\right), & \text{if } x \le 0, \\ 1-(1-\alpha) \cdot Q\left(\frac{1}{q_2}\left(\frac{\lvert x\rvert}{2-2\alpha^*}\right)^{q_2}, \frac{1}{q_2}\right), & \text{if } x > 0 \end{cases}$ | $\begin{cases} -2\alpha^*\left[q_1Q^{-1}\left(\frac{p}{\alpha}, \frac{1}{q_1}\right)\right]^{\frac{1}{q_1}}, & \text{if } p \le \alpha, \\ 2\left(1-\alpha^*\right) \cdot \left[q_2Q^{-1}\left(\frac{1-p}{1-\alpha}, \frac{1}{q_2}\right)\right]^{\frac{1}{q_2}}, & \text{if } p > \alpha \end{cases}$ | $\begin{cases} -\frac{2\alpha^*}{p}\int_0^p \left[q_1Q^{-1}\left(\frac{v}{\alpha}, \frac{1}{q_1}\right)\right]^{\frac{1}{q_1}} dv, & \text{if } p \le \alpha, \\ -\frac{2\alpha^*}{p}\int_0^\alpha \left[q_1Q^{-1}\left(\frac{v}{\alpha}, \frac{1}{q_1}\right)\right]^{\frac{1}{q_1}} dv + \frac{2(1-\alpha^*)}{p}\int_\alpha^p \left[q_2Q^{-1}\left(\frac{1-v}{1-\alpha}, \frac{1}{q_2}\right)\right]^{\frac{1}{q_2}} dv, & \text{if } p > \alpha \end{cases}$ |
| Beta normal | $\frac{\phi\left(\frac{x-\mu}{\sigma}\right)}{\sigma B(a,b)} \cdot \Phi^{a-1}\left(\frac{x-\mu}{\sigma}\right) \cdot \Phi^{b-1}\left(\frac{\mu-x}{\sigma}\right)$, $-\infty < x < \infty$ | $I_{\Phi\left(\frac{x-\mu}{\sigma}\right)}(a,b)$ | $\mu + \sigma\Phi^{-1}\left(I_p^{-1}(a,b)\right)$ | $\mu + \frac{\sigma}{p}\int_0^p \Phi^{-1}\left(I_v^{-1}(a,b)\right) dv$ |
| Half normal | $\frac{2}{\sigma}\phi\left(\frac{x}{\sigma}\right)$, $x > 0$ | $2\Phi\left(\frac{x}{\sigma}\right) - 1$ | $\sigma\Phi^{-1}\left(\frac{1+p}{2}\right)$ | $\frac{\sigma}{p}\int_0^p \Phi^{-1}\left(\frac{1+v}{2}\right) dv$ |
| Kumaraswamy half normal | $\frac{2ab}{\sigma}\phi\left(\frac{x}{\sigma}\right) \cdot \left[2\Phi\left(\frac{x}{\sigma}\right) - 1\right]^{a-1} \cdot \left\{1 - \left[2\Phi\left(\frac{x}{\sigma}\right) - 1\right]^{a}\right\}^{b-1}$, $x > 0$ | $1 - \left\{1 - \left[2\Phi\left(\frac{x}{\sigma}\right) - 1\right]^{a}\right\}^{b}$ | $\sigma\Phi^{-1}\left(\frac{1}{2} + \frac{1}{2}\left[1 - (1-p)^{1/b}\right]^{1/a}\right)$ | $\frac{\sigma}{p}\int_0^p \Phi^{-1}\left(\frac{1}{2} + \frac{1}{2}\left[1 - (1-v)^{1/b}\right]^{1/a}\right) dv$ |
| Student's $t$ | $\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)} \cdot \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$, $-\infty < x < \infty$ | $\frac{1+\text{sign}(x)}{2} - \frac{\text{sign}(x)}{2} I_{\frac{n}{x^2+n}}\left(\frac{n}{2}, \frac{1}{2}\right)$ | $\sqrt{n}\,\text{sign}\left(p - \frac{1}{2}\right) \cdot \sqrt{\frac{1}{I_a^{-1}\left(\frac{n}{2}, \frac{1}{2}\right)} - 1}$, where $a = 2p$ if $p < 1/2$, $a = 2(1-p)$ if $p \ge 1/2$ | $\frac{\sqrt{n}}{p}\int_0^p \text{sign}\left(v - \frac{1}{2}\right) \cdot \sqrt{\frac{1}{I_a^{-1}\left(\frac{n}{2}, \frac{1}{2}\right)} - 1}\, dv$, where $a = 2v$ if $v < 1/2$, $a = 2(1-v)$ if $v \ge 1/2$ |

| | | | | |
|---|---|---|---|---|
| Skewed Student's $t$ | $\begin{cases} K(\nu) \cdot \left[1+\frac{1}{\nu} \cdot \left(\frac{x}{2\alpha}\right)^2\right]^{-\frac{\nu+1}{2}}, & \text{if } x \le 0, \\ K(\nu) \cdot \left[1+\frac{1}{\nu} \cdot \left(\frac{x}{2(1-\alpha)}\right)^2\right]^{-\frac{\nu+1}{2}}, & \text{if } x > 0, \end{cases}$ where $K(\nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)}$ | $2\alpha F_\nu\left(\frac{\min(x,0)}{2\alpha}\right) -1+\alpha +2(1-\alpha)F_\nu\left(\frac{\max(x,0)}{2-2\alpha}\right)$, where $F_\nu(\cdot)$ is Student's $t$ cdf | $2\alpha F_\nu^{-1}\left(\frac{\min(p,\alpha)}{2\alpha}\right) +2(1-\alpha) \cdot F_\nu^{-1}\left(\frac{\max(p,\alpha)+1-2\alpha}{2-2\alpha}\right)$, where $F_\nu^{-1}(\cdot)$ is Student's $t$ inverse cdf | $\frac{2\alpha}{p}\int_0^p F_\nu^{-1}\left(\frac{\min(v,\alpha)}{2\alpha}\right) dv + \frac{2(1-\alpha)}{p}\int_0^p F_\nu^{-1}\left(\frac{\max(v,\alpha)+1-2\alpha}{2-2\alpha}\right) dv$ |
| Asymmetric Student's $t$ | $\begin{cases} \frac{\alpha}{\alpha^*} K(\nu_1) \cdot \left[1+\frac{1}{\nu_1} \cdot \left(\frac{x}{2\alpha^*}\right)^2\right]^{-\frac{\nu_1+1}{2}}, & \text{if } x \le 0, \\ \frac{1-\alpha}{1-\alpha^*} K(\nu_2) \cdot \left[1+\frac{1}{\nu_2} \cdot \left(\frac{x}{2(1-\alpha^*)}\right)^2\right]^{-\frac{\nu_2+1}{2}}, & \text{if } x > 0, \end{cases}$ where $K(\nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)}$, $\alpha^* = \frac{\alpha K(\nu_1)}{\alpha K(\nu_1)+(1-\alpha)K(\nu_2)}$ | $2\alpha F_{\nu_1}\left(\frac{\min(x,0)}{2\alpha^*}\right) -1+\alpha +2(1-\alpha)F_{\nu_2}\left(\frac{\max(x,0)}{2-2\alpha^*}\right)$, where $F_\nu(\cdot)$ is Student's $t$ cdf | $2\alpha^* F_{\nu_1}^{-1}\left(\frac{\min(p,\alpha)}{2\alpha}\right) +2\left(1-\alpha^*\right) \cdot F_{\nu_2}^{-1}\left(\frac{\max(p,\alpha)+1-2\alpha}{2-2\alpha}\right)$, where $F_\nu^{-1}(\cdot)$ is Student's $t$ inverse cdf | $\frac{2\alpha^*}{p}\int_0^p F_{\nu_1}^{-1}\left(\frac{\min(v,\alpha)}{2\alpha}\right) dv + \frac{2(1-\alpha^*)}{p}\int_0^p F_{\nu_2}^{-1}\left(\frac{\max(v,\alpha)+1-2\alpha}{2-2\alpha}\right) dv$ |
| Half Student's $t$ | $\frac{2\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)} \cdot \left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$, $x > 0$ | $I_{\frac{x^2}{x^2+n}}\left(\frac{1}{2}, \frac{n}{2}\right)$ | $\sqrt{\frac{n I_p^{-1}\left(\frac{1}{2}, \frac{n}{2}\right)}{1-I_p^{-1}\left(\frac{1}{2}, \frac{n}{2}\right)}}$ | $\frac{\sqrt{n}}{p}\int_0^p \sqrt{\frac{I_v^{-1}\left(\frac{1}{2}, \frac{n}{2}\right)}{1-I_v^{-1}\left(\frac{1}{2}, \frac{n}{2}\right)}} dv$ |
| Cauchy | $\frac{1}{\pi}\frac{\sigma}{(x-\mu)^2+\sigma^2}$, $-\infty < x < \infty$ | $\frac{1}{2}+\frac{1}{\pi}\arctan\left(\frac{x-\mu}{\sigma}\right)$ | $\mu+\sigma\tan\left(\pi\left(p-\frac{1}{2}\right)\right)$ | $\mu+\frac{\sigma}{p}\int_0^p \tan\left(\pi\left(v-\frac{1}{2}\right)\right) dv$ |
| Log Cauchy | $\frac{1}{x\pi}\frac{\sigma}{(\log x-\mu)^2+\sigma^2}$, $x > 0$ | $\frac{1}{\pi}\arctan\left(\frac{\log x-\mu}{\sigma}\right)$ | $\exp\left[\mu+\sigma\tan(\pi p)\right]$ | $\frac{\exp(\mu)}{p}\int_0^p \exp\left[\sigma\tan(\pi v)\right] dv$ |
| Half Cauchy | $\frac{2}{\pi}\frac{\sigma}{x^2+\sigma^2}$, $x > 0$ | $\frac{2}{\pi}\arctan\left(\frac{x}{\sigma}\right)$ | $\sigma\tan\left(\frac{\pi p}{2}\right)$ | $\frac{\sigma}{p}\int_0^p \tan\left(\frac{\pi v}{2}\right) dv$ |
| Laplace | $\frac{1}{2\sigma}\exp\left(-\frac{\lvert x-\mu\rvert}{\sigma}\right)$, $-\infty < x < \infty$ | $\begin{cases} \frac{1}{2}\exp\left(\frac{x-\mu}{\sigma}\right), & \text{if } x < \mu, \\ 1-\frac{1}{2}\exp\left(-\frac{x-\mu}{\sigma}\right), & \text{if } x \ge \mu \end{cases}$ | $\begin{cases} \mu+\sigma\log(2p), & \text{if } p < 1/2, \\ \mu-\sigma\log\left[2(1-p)\right], & \text{if } p \ge 1/2 \end{cases}$ | $\begin{cases} \mu+\sigma\left[\log(2p)-1\right], & \text{if } p < 1/2, \\ \mu+\sigma-\frac{\sigma}{p} +\sigma\frac{1-p}{p}\log(1-p) +\sigma\frac{1-p}{p}\log 2, & \text{if } p \ge 1/2 \end{cases}$ |
| Poiraud-Casanova-Thomas-Agnan Laplace | $\begin{cases} \alpha(1-\alpha) \cdot \exp\left\{(1-\alpha)(x-\theta)\right\}, & \text{if } x \le \theta, \\ \alpha(1-\alpha) \cdot \exp\left\{\alpha(\theta-x)\right\}, & \text{if } x > \theta \end{cases}$ | $\begin{cases} \alpha\exp\left\{(1-\alpha)(x-\theta)\right\}, & \text{if } x \le \theta, \\ 1-(1-\alpha) \cdot \exp\left\{\alpha(\theta-x)\right\}, & \text{if } x > \theta \end{cases}$ | $\begin{cases} \theta+\frac{1}{1-\alpha} \cdot \log\left(\frac{p}{\alpha}\right), & \text{if } p \le \alpha, \\ \theta-\frac{1}{\alpha} \cdot \log\left(\frac{1-p}{1-\alpha}\right), & \text{if } p > \alpha \end{cases}$ | $\begin{cases} \theta-\frac{\log\alpha}{1-\alpha} +\frac{\log p-1}{(1-\alpha)p}, & \text{if } p \le \alpha, \\ \theta-\frac{1}{\alpha} +\frac{1}{p}-\frac{\alpha}{(1-\alpha)p} +\frac{1-p}{\alpha p} \cdot \log\left(\frac{1-p}{1-\alpha}\right), & \text{if } p > \alpha \end{cases}$ |

| | | | | |
|---|---|---|---|---|
| Holla-Bhattacharya Laplace | $\begin{cases} a\phi \cdot \exp\left\{\phi\left(x-\theta\right)\right\}, & \text{if } x \leq \theta, \\ \left(1-a\right)\phi \cdot \exp\left\{\phi\left(\theta-x\right)\right\}, & \text{if } x > \theta \end{cases}$ | $\begin{cases} a\exp\left(-\theta\phi\right) \cdot \exp\left(\phi x\right), & \text{if } x \leq \theta, \\ 1-\left(1-a\right)\exp\left(\theta\phi\right) \cdot \exp\left(-\phi x\right), & \text{if } x > \theta \end{cases}$ | $\begin{cases} \theta + \dfrac{1}{\phi}\log\dfrac{p}{a}, & \text{if } p \leq a, \\ \theta - \dfrac{1}{\phi}\log\dfrac{1-p}{1-a}, & \text{if } p > a \end{cases}$ | $\begin{cases} \theta - \dfrac{1}{\phi} + \frac{1}{\phi}\log\frac{p}{a}, & \text{if } p \leq a, \\ \dfrac{1}{p}\left[\theta(1+p-a) + \frac{p-2a-(1-a)\log a}{\phi} + \frac{1-p}{\phi}\log\frac{1-p}{1-a}\right], & \text{if } p > a \end{cases}$ |
| McGill Laplace | $\begin{cases} \dfrac{1}{2\psi}\exp\left(\dfrac{x-\theta}{\psi}\right), & \text{if } x \leq \theta, \\ \dfrac{1}{2\phi}\exp\left(\dfrac{\theta-x}{\phi}\right), & \text{if } x > \theta \end{cases}$ | $\begin{cases} \dfrac{1}{2}\exp\left(\dfrac{x-\theta}{\psi}\right), & \text{if } x \leq \theta, \\ 1-\dfrac{1}{2}\exp\left(\dfrac{\theta-x}{\phi}\right), & \text{if } x > \theta \end{cases}$ | $\begin{cases} \theta + \psi\log(2p), & \text{if } p \leq 1/2, \\ \theta - \phi\log\left(2(1-p)\right), & \text{if } p > 1/2 \end{cases}$ | $\begin{cases} \psi + \theta\log(2p) - \theta p, & \text{if } p \leq 1/2, \\ \theta + \phi + \frac{\psi-\phi-2\theta}{2p} + \frac{\phi}{p}\log 2 - \phi\log 2 + \frac{\phi}{p}\log(1-p) - \phi\log(1-p), & \text{if } p > 1/2 \end{cases}$ |
| Log Laplace | $\begin{cases} \frac{\alpha\beta x^{\beta-1}}{\delta^{\beta}(\alpha+\beta)}, & \text{if } x \leq \delta, \\ \frac{\alpha\beta\delta^{\alpha}}{x^{\alpha+1}(\alpha+\beta)}, & \text{if } x > \delta \end{cases}$ | $\begin{cases} \frac{\alpha x^{\beta}}{\delta^{\beta}(\alpha+\beta)}, & \text{if } x \leq \delta, \\ 1-\frac{\beta\delta^{\alpha}}{x^{\alpha}(\alpha+\beta)}, & \text{if } x > \delta \end{cases}$ | $\begin{cases} \delta\left[p\frac{\alpha+\beta}{\alpha}\right]^{1/\beta}, & \text{if } p \leq \frac{\alpha}{\alpha+\beta}, \\ \delta\left[(1-p)\frac{\alpha+\beta}{\alpha}\right]^{-1/\alpha}, & \text{if } p > \frac{\alpha}{\alpha+\beta} \end{cases}$ | $\begin{cases} \frac{\delta\beta}{\beta+1}\left[p\frac{\alpha+\beta}{\alpha}\right]^{1/\beta}, & \text{if } p \leq \frac{\alpha}{\alpha+\beta}, \\ \frac{\alpha\delta}{p(1+1/\beta)(\alpha+\beta)} + \frac{\alpha^{1/\alpha}\beta^{1-1/\alpha}\delta}{p(\alpha+\beta)(1-1/\alpha)} - \frac{\delta(1-p)}{p(1-1/\alpha)} \cdot \left[\frac{\alpha}{(\alpha+\beta)(1-p)}\right]^{1/\alpha}, & \text{if } p > \frac{\alpha}{\alpha+\beta} \end{cases}$ |
| Asymmetric Laplace | $\begin{cases} \frac{\kappa\sqrt{2}}{\tau\left(1+\kappa^2\right)} \cdot \exp\left(-\frac{\kappa\sqrt{2}}{\tau}\lvert x-\theta\rvert\right), & \text{if } x \geq \theta, \\ \frac{\kappa\sqrt{2}}{\tau\left(1+\kappa^2\right)} \cdot \exp\left(-\frac{\sqrt{2}}{\kappa\tau}\lvert x-\theta\rvert\right), & \text{if } x < \theta \end{cases}$ | $\begin{cases} 1-\frac{1}{1+\kappa^2} \cdot \exp\left(\frac{\kappa\sqrt{2}(\theta-x)}{\tau}\right), & \text{if } x \geq \theta, \\ \frac{\kappa^2}{1+\kappa^2} \cdot \exp\left(\frac{\sqrt{2}(x-\theta)}{\kappa\tau}\right), & \text{if } x < \theta \end{cases}$ | $\begin{cases} \theta - \frac{\tau}{\sqrt{2}\kappa} \cdot \log\left[(1-p)\left(1+\kappa^2\right)\right], & \text{if } p \geq \frac{\kappa^2}{1+\kappa^2}, \\ \theta + \frac{\kappa\tau}{\sqrt{2}} \cdot \log\left[p\left(1+\kappa^{-2}\right)\right], & \text{if } p < \frac{\kappa^2}{1+\kappa^2} \end{cases}$ | $\begin{cases} \frac{\theta}{p} + \theta - \frac{\tau}{\sqrt{2}\kappa} \cdot \log\left(1+\kappa^2\right) + \frac{\sqrt{2}\tau\left(1+2\kappa^2\right)}{2\kappa\left(1+\kappa^2\right)p} \cdot \log\left(1+\kappa^2\right) - \frac{\sqrt{2}\tau\kappa\log\kappa}{\left(1+\kappa^2\right)p} - \frac{\theta\kappa^2}{\left(1+\kappa^2\right)p} + \frac{\tau\left(1-\kappa^4\right)}{\sqrt{2}\kappa\left(1+\kappa^2\right)p} - \frac{\tau(1-p)}{\sqrt{2}\kappa p} + \frac{\tau(1-p)}{\sqrt{2}\kappa p} \cdot \log(1-p), & \text{if } p \geq \frac{\kappa^2}{1+\kappa^2}, \\ \theta + \frac{\kappa\tau}{\sqrt{2}} \cdot \log\left(1+\kappa^{-2}\right) + \frac{\kappa\tau}{\sqrt{2}}(\log p - 1), & \text{if } p < \frac{\kappa^2}{1+\kappa^2} \end{cases}$ |

| | | | | |
|---|---|---|---|---|
| Asymmetric power | $\begin{cases} \frac{\delta^{1/\lambda}}{\Gamma(1+1/\lambda)} \cdot \exp\left[-\frac{\delta}{\alpha^\lambda}\lvert x\rvert^\lambda\right], & \text{if } x \le 0, \\ \frac{\delta^{1/\lambda}}{\Gamma(1+1/\lambda)} \cdot \exp\left[-\frac{\delta}{(1-\alpha)^\lambda}\lvert x\rvert^\lambda\right], & \text{if } x > 0 \end{cases}$ | $\begin{cases} \alpha - \alpha \cdot \mathcal{I}\left(\frac{\delta}{\alpha^\lambda}\sqrt{\lambda}\lvert x\rvert^\lambda, 1/\lambda\right), & \text{if } x \le 0, \\ \alpha - (1-\alpha) \cdot \mathcal{I}\left(\frac{\delta}{(1-\alpha)^\lambda}\sqrt{\lambda}\lvert x\rvert^\lambda, 1/\lambda\right), & \text{if } x > 0 \end{cases}$ | $\begin{cases} -\left[\frac{\alpha^\lambda}{\delta\sqrt{\lambda}}\right]^{1/\lambda} \cdot \left[\mathcal{I}^{-1}\left(1-\frac{p}{\alpha}, \frac{1}{\lambda}\right)\right]^{1/\lambda}, & \text{if } p \le \alpha, \\ -\left[\frac{(1-\alpha)^\lambda}{\delta\sqrt{\lambda}}\right]^{1/\lambda} \cdot \left[\mathcal{I}^{-1}\left(1-\frac{1-p}{1-\alpha}, \frac{1}{\lambda}\right)\right]^{1/\lambda}, & \text{if } p > \alpha \end{cases}$ | $\begin{cases} -\frac{1}{p}\left[\frac{\alpha^\lambda}{\delta\sqrt{\lambda}}\right]^{1/\lambda} \cdot \int_0^p \left[\mathcal{I}^{-1}\left(1-\frac{v}{\alpha}, \frac{1}{\lambda}\right)\right]^{1/\lambda} dv, & \text{if } p \le \alpha, \\ -\frac{1}{p}\left[\frac{\alpha^\lambda}{\delta\sqrt{\lambda}}\right]^{1/\lambda} \cdot \int_0^\alpha \left[\mathcal{I}^{-1}\left(1-\frac{v}{\alpha}, \frac{1}{\lambda}\right)\right]^{1/\lambda} dv - \frac{1}{p}\left[\frac{(1-\alpha)^\lambda}{\delta\sqrt{\lambda}}\right]^{1/\lambda} \cdot \int_\alpha^p \left[\mathcal{I}^{-1}\left(1-\frac{1-v}{1-\alpha}, \frac{1}{\lambda}\right)\right]^{1/\lambda} dv, & \text{if } p > \alpha \end{cases}$ |
| Logistic | $\frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right) \cdot \left[1+\exp\left(-\frac{x-\mu}{\sigma}\right)\right]^{-2}$, $-\infty < x < \infty$ | $\frac{1}{1+\exp\left(-\frac{x-\mu}{\sigma}\right)}$ | $\mu + \sigma\log\left[p(1-p)\right]$ | $\mu - 2\sigma + \sigma\log p - \sigma\frac{1-p}{p}\log(1-p)$ |
| Hyperbolic secant | $\frac{1}{2}\text{sech}\left(\frac{\pi x}{2}\right)$, $-\infty < x < \infty$ | $\frac{2}{\pi}\arctan\left[\exp\left(\frac{\pi x}{2}\right)\right]$ | $\frac{2}{\pi}\log\left[\tan\left(\frac{\pi p}{2}\right)\right]$ | $\frac{2}{\pi p}\int_0^p \log\left[\tan\left(\frac{\pi v}{2}\right)\right] dv$ |
| Generalized logistic | $\frac{a\exp\left(-\frac{x-\mu}{\theta}\right)}{\theta\left\{1+\exp\left(-\frac{x-\mu}{\theta}\right)\right\}^{1+a}}$, $-\infty < x < \infty$ | $\frac{1}{\left\{1+\exp\left(-\frac{x-\mu}{\theta}\right)\right\}^a}$ | $\mu - \theta\log\left(p^{-1/a}-1\right)$ | $\mu - \frac{\theta}{p}\int_0^p \log\left(v^{-1/a}-1\right) dv$ |
| Generalized logistic III | $\frac{1}{\theta B(\alpha,\alpha)}\exp\left(\alpha\frac{x-\mu}{\theta}\right) \cdot \left\{1+\exp\left(\frac{x-\mu}{\theta}\right)\right\}^{-2\alpha}$, $-\infty < x < \infty$ | $I_{\frac{1}{1+\exp\left(-\frac{x-\mu}{\theta}\right)}}(\alpha,\alpha)$ | $\mu - \theta\log\frac{1-I_p^{-1}(\alpha,\alpha)}{I_p^{-1}(\alpha,\alpha)}$ | $\mu - \frac{\theta}{p}\int_0^p \log\frac{1-I_v^{-1}(\alpha,\alpha)}{I_v^{-1}(\alpha,\alpha)} dv$ |
| Generalized logistic IV | $\frac{1}{\theta B(\alpha,a)}\exp\left(-\alpha\frac{x-\mu}{\theta}\right) \cdot \left\{1+\exp\left(-\frac{x-\mu}{\theta}\right)\right\}^{-\alpha-a}$, $-\infty < x < \infty$ | $I_{\frac{1}{1+\exp\left(-\frac{x-\mu}{\theta}\right)}}(\alpha,a)$ | $\mu - \theta\log\frac{1-I_p^{-1}(\alpha,a)}{I_p^{-1}(\alpha,a)}$ | $\mu - \frac{\theta}{p}\int_0^p \log\frac{1-I_v^{-1}(\alpha,a)}{I_v^{-1}(\alpha,a)} dv$ |
| Half logistic | $\frac{2\lambda\exp(-\lambda x)}{[1+\exp(-\lambda x)]^2}$, $x > 0$ | $\frac{1-\exp(-\lambda x)}{1+\exp(-\lambda x)}$ | $-\frac{1}{\lambda}\log\frac{1-p}{1+p}$ | $-\frac{1}{\lambda}\log\frac{1-p}{1+p} + \frac{1}{\lambda p}\log\left(1-p^2\right)$ |
| Log-logistic | $\frac{\beta\alpha^\beta x^{\beta-1}}{\left(\alpha^\beta+x^\beta\right)^2}$, $x > 0$ | $\frac{x^\beta}{\alpha^\beta+x^\beta}$ | $\alpha\left(\frac{p}{1-p}\right)^{1/\beta}$ | $\frac{\alpha}{p}B_p\left(1+\frac{1}{\beta}, 1-\frac{1}{\beta}\right)$ |
| Kumaraswamy log-logistic | $\frac{ab\beta\alpha^\beta x^{a\beta-1}}{\left(\alpha^\beta+x^\beta\right)^{a+1}} \cdot \left[1-\frac{x^{a\beta}}{\left(\alpha^\beta+x^\beta\right)^a}\right]^{b-1}$, $x > 0$ | $\left[1-\frac{x^{a\beta}}{\left(\alpha^\beta+x^\beta\right)^a}\right]^b$ | $\alpha\left\{\left[1-(1-p)^{1/b}\right]^{1/a}-1\right\}^{-1/\beta}$ | $\frac{\alpha}{p}\int_0^p \left\{\left[1-(1-v)^{1/b}\right]^{1/a}-1\right\}^{-1/\beta} dv$ |
| Exponentiated logistic | $(\alpha/\beta)\exp(-x/\beta) \cdot [1+\exp(-x/\beta)]^{-\alpha-1}$, $-\infty < x < \infty$ | $[1+\exp(-x/\beta)]^{-\alpha}$ | $-\beta\log\left[p^{-1/\alpha}-1\right]$ | $-\frac{\beta}{p}\int_0^p \log\left[v^{-1/\alpha}-1\right] dv$ |
| Hosking logistic | $\frac{(1-kx)^{1/k-1}}{\left[1+(1-kx)^{1/k}\right]^2}$, $x < 1/k$ if $k > 0$, $x > 1/k$ if $k < 0$, $-\infty < x < \infty$ if $k = 0$ | $\frac{1}{1+(1-kx)^{1/k}}$ | $\frac{1}{k}\left[1-\left(\frac{1-p}{p}\right)^k\right]$ | $\frac{1}{k} - \frac{1}{kp}B_p(1-k, 1+k)$ |
| lognormal | $\frac{1}{\sigma x}\phi\left(\frac{\log x-\mu}{\sigma}\right)$, $x > 0$ | $\Phi\left(\frac{\log x-\mu}{\sigma}\right)$ | $\exp\left[\mu+\sigma\Phi^{-1}(p)\right]$ | $\frac{\exp(\mu)}{p}\int_0^p \exp\left[\sigma\Phi^{-1}(v)\right] dv$ |

| | | | | |
|---|---|---|---|---|
| Beta lognormal | $\frac{1}{\sigma x B(a,b)} \cdot \phi\left(\frac{\log x-\mu}{\sigma}\right) \cdot \Phi^{a-1}\left(\frac{\log x-\mu}{\sigma}\right) \cdot \Phi^{b-1}\left(\frac{\mu-\log x}{\sigma}\right)$, $x>0$ | $I_{\Phi\left(\frac{\log x-\mu}{\sigma}\right)}(a,b)$ | $\exp\left[\mu+\sigma \cdot \Phi^{-1}\left(I_p^{-1}(a,b)\right)\right]$ | $\frac{\exp(\mu)}{p}\int_0^p \exp\left[\sigma \cdot \Phi^{-1}\left(I_v^{-1}(a,b)\right)\right] dv$ |
| Burr | $\frac{ba^b}{x^{b+1}} \cdot \left[1+(x/a)^{-b}\right]^{-2}$, $x>0$ | $\frac{1}{1+(x/a)^{-b}}$ | $ap^{1/b}(1-p)^{-1/b}$ | $\frac{a}{p} B_p\left(1/b+1, 1-1/b\right)$ |
| Beta Burr | $\frac{ba^{bd}}{B(c,d)x^{bd+1}} \cdot \left[1+(x/a)^{-b}\right]^{-c-d}$, $x>0$ | $I_{\frac{1}{1+(x/a)^{-b}}}(c,d)$ | $a\left[I_p^{-1}(c,d)\right]^{1/b} \cdot \left[1-I_p^{-1}(c,d)\right]^{-1/b}$ | $\frac{a}{p}\int_0^p \left[I_v^{-1}(c,d)\right]^{1/b} \cdot \left[1-I_v^{-1}(c,d)\right]^{-1/b} dv$ |
| Burr XII | $\frac{kcx^{c-1}}{(1+x^c)^{k+1}}$, $x>0$ | $1-(1+x^c)^{-k}$ | $\left[(1-p)^{-1/k}-1\right]^{1/c}$ | $\frac{1}{p}\int_0^p \left[(1-v)^{-1/k}-1\right]^{1/c} dv$ |
| Kumaraswamy Burr XII | $\frac{abkcx^{c-1}}{(1+x^c)^{k+1}} \cdot \left[1-(1+x^c)^{-k}\right]^{a-1} \cdot \left\{1-\left[1-\left(1+x^c\right)^{-k}\right]^a\right\}^{b-1}$, $x>0$ | $1-\left\{1-\left[1-\left(1+x^c\right)^{-k}\right]^a\right\}^b$ | $\left[\left\{1-\left[1-(1-p)^{1/b}\right]^{1/a}\right\}^{-1/k}-1\right]^{1/c}$ | $\frac{1}{p}\int_0^p \left[\left\{1-\left[1-(1-v)^{1/b}\right]^{1/a}\right\}^{-1/k}-1\right]^{1/c} dv$ |
| Beta Burr XII | $\frac{kcx^{c-1}}{B(a,b)} \cdot \left[1-(1+x^c)^{-k}\right]^{a-1} \cdot (1+x^c)^{-bk-1}$, $x>0$ | $I_{1-(1+x^c)^{-k}}(a,b)$ | $\left\{\left[1-I_p^{-1}(a,b)\right]^{-1/k}-1\right\}^{1/c}$ | $\frac{1}{p}\int_0^p \left\{\left[1-I_v^{-1}(a,b)\right]^{-1/k}-1\right\}^{1/c} dv$ |
| Dagum | $\frac{acb^a x^{ac-1}}{[x^a+b^a]^{c+1}}$, $x>0$ | $\left[1+\left(\frac{b}{x}\right)^a\right]^{-c}$ | $b\left(1-p^{-1/c}\right)^{-1/a}$ | $\frac{b}{p}\int_0^p \left(1-v^{-1/c}\right)^{-1/a} dv$ |
| Lomax | $\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-\alpha-1}$, $x>0$ | $1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}$ | $\lambda\left[(1-p)^{-1/\alpha}-1\right]$ | $-\lambda + \frac{\lambda-\lambda(1-p)^{1-1/\alpha}}{p-p/\alpha}$ |
| Beta Lomax | $\frac{\alpha}{\lambda B(a,b)} \cdot \left(1+\frac{x}{\lambda}\right)^{-b\alpha-1} \cdot \left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{a-1}$, $x>0$ | $I_{1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}}(a,b)$ | $\lambda\left[1-I_p^{-1}(a,b)\right]^{-1/\alpha} - \lambda$ | $\frac{\lambda}{p}\int_0^p \left[1-I_v^{-1}(a,b)\right]^{-1/\alpha} dv - \lambda$ |
| Gumbel | $\exp(-x) \cdot \exp\left[-\exp(-x)\right]$, $-\infty<x<\infty$ | $\exp\left[-\exp(-x)\right]$ | $-\log(-\log p)$ | $-\frac{1}{p}\int_0^p \log(-\log v) dv$ |
| Kumaraswamy Gumbel | $ab\exp(-x) \cdot \exp\left[-a\exp(-x)\right] \cdot \left\{1-\exp\left[-a\exp(-x)\right]\right\}^{b-1}$, $-\infty<x<\infty$ | $1-\left\{1-\exp\left[-a\exp(-x)\right]\right\}^b$, $-\infty<x<\infty$ | $-\log\left\{-\log\left[1-(1-p)^{1/b}\right]^{1/a}\right\}$ | $-\frac{1}{p}\int_0^p \log\left\{-\log\left[1-(1-v)^{1/b}\right]^{1/a}\right\} dv$ |
| Beta Gumbel | $\frac{\exp(-x)}{B(a,b)} \cdot \exp\left[-a\exp(-x)\right] \cdot \left\{1-\exp\left[-\exp(-x)\right]\right\}^{b-1}$, $-\infty<x<\infty$ | $I_{\exp[-\exp(-x)]}(a,b)$ | $-\log\left[-\log I_p^{-1}(a,b)\right]$ | $-\frac{1}{p}\int_0^p \log\left[-\log I_v^{-1}(a,b)\right] dv$ |
| Gumbel II | $abx^{-a-1}\exp\left(-bx^{-a}\right)$, $x>0$ | $1-\exp\left(-bx^{-a}\right)$ | $b^{1/a}\left[-\log(1-p)\right]^{-1/a}$ | $\frac{b^{1/a}}{p}\int_0^p \left[-\log(1-v)\right]^{-1/a} dv$ |
| Beta Gumbel II | $\frac{abx^{-a-1}}{B(c,d)} \cdot \exp\left(-bdx^{-a}\right) \cdot \left[1-\exp\left(-bx^{-a}\right)\right]^{c-1}$, $x>0$ | $I_{1-\exp\left(-bx^{-a}\right)}(c,d)$ | $b^{1/a}\left\{-\log\left[1-I_p^{-1}(c,d)\right]\right\}^{-1/a}$ | $\frac{b^{1/a}}{p}\int_0^p \left\{-\log\left[1-I_v^{-1}(c,d)\right]\right\}^{-1/a} dv$ |

| | | | | |
|---|---|---|---|---|
| Fréchet | $\frac{\alpha\sigma^{\alpha}}{x^{\alpha+1}} \cdot \exp\left\{-\left(\frac{\sigma}{x}\right)^{\alpha}\right\}$, $x>0$ | $\exp\left\{-\left(\frac{\sigma}{x}\right)^{\alpha}\right\}$ | $\sigma\left[-\log p\right]^{-1/\alpha}$ | $\frac{\sigma}{p}\Gamma\left(1-1/\alpha, -\log p\right)$ |
| Beta Fréchet | $\frac{\alpha\sigma^{\alpha}}{x^{\alpha+1}B(a,b)} \cdot \exp\left\{-a\left(\frac{\sigma}{x}\right)^{\alpha}\right\} \cdot \left[1-\exp\left\{-\left(\frac{\sigma}{x}\right)^{\alpha}\right\}\right]^{b-1}$, $x>0$ | $I_{\exp\left\{-\left(\frac{\sigma}{x}\right)^{\alpha}\right\}}(a,b)$ | $\sigma\left[-\log I_p^{-1}(a,b)\right]^{-1/\alpha}$ | $\frac{\sigma}{p}\int_0^p\left[-\log I_v^{-1}(a,b)\right]^{-1/\alpha}dv$ |
| Weibull | $\frac{\alpha x^{\alpha-1}}{\sigma^{\alpha}} \cdot \exp\left\{-\left(\frac{x}{\sigma}\right)^{\alpha}\right\}$, $x>0$ | $1-\exp\left\{-\left(\frac{x}{\sigma}\right)^{\alpha}\right\}$ | $\sigma\left[-\log(1-p)\right]^{1/\alpha}$ | $\frac{\sigma}{p}\gamma\left(1+1/\alpha, -\log(1-p)\right)$ |
| Kumaraswamy Weibull | $\frac{ab\alpha x^{\alpha-1}}{\sigma^{\alpha}}\exp\left[-\left(\frac{x}{\sigma}\right)^{\alpha}\right] \cdot \left\{1-\exp\left[-\left(\frac{x}{\sigma}\right)^{\alpha}\right]\right\}^{a-1} \cdot \left[1-\left\{1-\exp\left[-\left(\frac{x}{\sigma}\right)^{\alpha}\right]\right\}^{a}\right]^{b-1}$, $x>0$ | $1-\left[1-\left\{1-\exp\left[-\left(\frac{x}{\sigma}\right)^{\alpha}\right]\right\}^{a}\right]^{b}$ | $\sigma\left[-\log\left\{1-\left[1-(1-p)^{1/b}\right]^{1/a}\right\}\right]^{1/\alpha}$ | $\frac{\sigma}{p}\int_0^p\left[-\log\left\{1-\left[1-(1-v)^{1/b}\right]^{1/a}\right\}\right]^{1/\alpha}dv$ |
| Logistic Rayleigh | $\alpha\lambda x\exp\left(\lambda x^2/2\right) \cdot \left[\exp\left(\lambda x^2/2\right)-1\right]^{\alpha-1} \cdot \left\{1+\left[\exp\left(\lambda x^2/2\right)-1\right]^{\alpha}\right\}^{-2}$, $x>0$ | $\frac{\left[\exp\left(\lambda x^2/2\right)-1\right]^{\alpha}}{1+\left[\exp\left(\lambda x^2/2\right)-1\right]^{\alpha}}$ | $\sqrt{\frac{2}{\lambda}} \cdot \sqrt{\log\left[1+\left(\frac{p}{1-p}\right)^{1/\alpha}\right]}$ | $\frac{\sqrt{2}}{p\sqrt{\lambda}}\int_0^p\left\{\log\left[1+\left(\frac{v}{1-v}\right)^{1/\alpha}\right]\right\}^{1/2}dv$ |
| Marshall-Olkin Weibull | $\beta\lambda^{\beta}x^{\beta-1}\exp\left[(\lambda x)^{\beta}\right] \cdot \left\{\exp\left[(\lambda x)^{\beta}\right]-1+\alpha\right\}^{-2}$, $x>0$ | $\frac{\exp\left[(\lambda x)^{\beta}\right]-2+\alpha}{\exp\left[(\lambda x)^{\beta}\right]-1+\alpha}$ | $\frac{1}{\lambda}\left[\log\left(\frac{1}{1-p}+1-\alpha\right)\right]^{1/\beta}$ | $\frac{1}{\lambda p}\int_0^p\left[\log\left(\frac{1}{1-v}+1-\alpha\right)\right]^{1/\beta}dv$ |
| Beta Weibull | $\frac{\alpha x^{\alpha-1}}{\sigma^{\alpha}B(a,b)} \cdot \exp\left\{-b\left(\frac{x}{\sigma}\right)^{\alpha}\right\} \cdot \left[1-\exp\left\{-\left(\frac{x}{\sigma}\right)^{\alpha}\right\}\right]^{a-1}$, $x>0$ | $I_{1-\exp\left\{-\left(\frac{x}{\sigma}\right)^{\alpha}\right\}}(a,b)$ | $\sigma\left\{-\log\left[1-I_p^{-1}(a,b)\right]\right\}^{1/\alpha}$ | $\frac{\sigma}{p}\int_0^p\left\{-\log\left[1-I_v^{-1}(a,b)\right]\right\}^{1/\alpha}dv$ |
| Double Weibull | $\frac{c}{2\phi}\left\lvert\frac{x-\theta}{\phi}\right\rvert^{c-1} \cdot \exp\left\{-\left\lvert\frac{x-\theta}{\phi}\right\rvert^{c}\right\}$, $-\infty<x<\infty$ | $\begin{cases}\frac{1}{2}\exp\left\{-\left(\frac{\theta-x}{\phi}\right)^{c}\right\}, & \text{if } x\leq\theta, \\ 1-\frac{1}{2}\exp\left\{-\left(\frac{x-\theta}{\phi}\right)^{c}\right\}, & \text{if } x>\theta\end{cases}$ | $\begin{cases}\theta-\phi\cdot\left[-\log(2p)\right]^{1/c}, & \text{if } p\leq 1/2, \\ \theta+\phi\cdot\left[-\log(2(1-p))\right]^{1/c}, & \text{if } p>1/2\end{cases}$ | $\begin{cases}\theta-\frac{\phi}{p}\cdot\int_0^p\left[-\log 2-\log v\right]^{1/c}dv, & \text{if } p\leq 1/2, \\ \theta-\frac{\phi}{p}\cdot\int_0^{1/2}\left[-\log 2-\log v\right]^{1/c}dv+\frac{\phi}{p}\cdot\int_{1/2}^p\left[-\log 2-\log(1-v)\right]^{1/c}dv, & \text{if } p>1/2\end{cases}$ |
| Exponentiated Weibull | $c\alpha\lambda^{-c}x^{c-1} \cdot \exp\left[-(x/\lambda)^{c}\right] \cdot \left\{1-\exp\left[-(x/\lambda)^{c}\right]\right\}^{\alpha-1}$, $x>0$ | $\left\{1-\exp\left[-(x/\lambda)^{c}\right]\right\}^{\alpha}$ | $\lambda\left[-\log\left(1-p^{1/\alpha}\right)\right]^{1/c}$ | $\frac{\lambda}{p}\int_0^p\left[-\log\left(1-v^{1/\alpha}\right)\right]^{1/c}dv$ |
| Generalized power Weibull | $\alpha\theta x^{\alpha-1}\left[1+x^{\alpha}\right]^{\theta-1} \cdot \exp\left\{1-\left[1+x^{\alpha}\right]^{\theta}\right\}$, $x>0$ | $1-\exp\left\{1-\left[1+x^{\alpha}\right]^{\theta}\right\}$ | $\left\{\left[1-\log(1-p)\right]^{1/\theta}-1\right\}^{1/\alpha}$ | $\frac{1}{p}\int_0^p\left\{\left[1-\log(1-v)\right]^{1/\theta}-1\right\}^{1/\alpha}dv$ |

| | | | | |
|---|---|---|---|---|
| Generalized inverse Weibull | $a^b bcx^{-b-1} \cdot \exp\left[-c\left(\frac{a}{x}\right)^b\right]$, $x>0$ | $\exp\left[-c\left(\frac{a}{x}\right)^b\right]$ | $ac^{1/b}(-\log p)^{-1/b}$ | $\frac{ac^{1/b}}{p} \cdot \int_0^p (-\log v)^{-1/b} dv$ |
| Beta generalized Rayleigh | $\frac{2\theta^{\alpha+1} x^{2\alpha+1}}{B(a,b)\Gamma(\alpha+1)} \cdot \exp\left(-\theta x^2\right) \cdot \left[\frac{\gamma\left(\alpha+1,\theta x^2\right)}{\Gamma(\alpha+1)}\right]^{a-1} \cdot Q^{b-1}\left(\alpha+1,\theta x^2\right)$, $x>0$ | $I_{1-Q\left(\alpha+1,\theta x^2\right)}(a,b)$ | $\sqrt{\frac{1}{\theta} Q^{-1}\left[1-I_p^{-1}(a,b)\right]}$ | $\frac{1}{\sqrt{\theta}} \int_0^p \left\{Q^{-1}\left[1 - I_v^{-1}(a,b)\right]\right\}^{1/2} dv$ |
| Chen | $\lambda\beta x^{\beta-1}\exp\left(x^\beta\right) \cdot \exp\left[\lambda-\lambda\exp\left(x^\beta\right)\right]$, $x>0$ | $1-\exp\left[\lambda-\lambda\exp\left(x^\beta\right)\right]$ | $\left\{\log\left[1-\frac{\log(1-p)}{\lambda}\right]\right\}^{1/\beta}$ | $\frac{1}{p}\int_0^p \left\{\log\left[1 - \frac{\log(1-v)}{\lambda}\right]\right\}^{1/\beta} dv$ |
| Xie | $\lambda\beta\left(\frac{x}{\alpha}\right)^{\beta-1} \cdot \exp\left[(x/\alpha)^\beta\right] \cdot \exp(\lambda\alpha) \cdot \exp\left\{-\lambda\alpha\exp\left[(x/\alpha)^\beta\right]\right\}$, $x>0$ | $1-\exp(\lambda\alpha) \cdot \exp\left\{-\lambda\alpha\exp\left[(x/\alpha)^\beta\right]\right\}$ | $\alpha\left\{\log\left[1-\frac{\log(1-p)}{\lambda\alpha}\right]\right\}^{1/\beta}$ | $\frac{\alpha}{p}\int_0^p \left\{\log\left[1 - \frac{\log(1-v)}{\lambda\alpha}\right]\right\}^{1/\beta} dv$ |
| Tukey Lambda | | | $\frac{p^\lambda-(1-p)^\lambda}{\lambda}$ | $\frac{p^{\lambda+1}+(1-p)^{\lambda+1}-1}{p\lambda(\lambda+1)}$ |
| Govindarajulu | | | $\theta + \sigma(\beta+1)p^\beta - \sigma\beta p^{\beta+1}$ | $\theta + \sigma p^\beta - \frac{\sigma\beta}{\beta+2}p^{\beta+2}$ |
| Ramberg-Schmeiser | | | $\frac{p^\beta-(1-p)^\gamma}{\delta}$ | $\frac{p^\beta}{\delta(\beta+1)} + \frac{(1-p)^{\gamma+1}-1}{p\delta(\gamma+1)}$ |
| Freimer | | | $\frac{1}{\alpha}\left[\frac{p^\beta-1}{\beta}-\frac{(1-p)^\gamma-1}{\gamma}\right]$ | $\frac{1}{\alpha}\left(\frac{1}{\gamma}-\frac{1}{\beta}\right) + \frac{p^\beta}{\alpha\beta(\beta+1)} + \frac{(1-p)^{\gamma+1}-1}{p\alpha\gamma(\gamma+1)}$ |
| Hankin-Lee | | | $\frac{Cp^\alpha}{(1-p)^\beta}$ | $\frac{C}{p}B_p(\alpha+1,1-\beta)$ |
| van Staden-Loots | | | $\lambda_1 - \frac{\lambda_2(1-\lambda_3)}{\lambda_4} + \frac{\lambda_2\lambda_3}{\lambda_4} + \frac{\lambda_2(1-\lambda_3)}{\lambda_4}p^{\lambda_4} - \frac{\lambda_2\lambda_3}{\lambda_4}(1-p)^{\lambda_4}$ | $\lambda_1 - \frac{\lambda_2(1-\lambda_3)}{\lambda_4} + \frac{\lambda_2\lambda_3}{\lambda_4} + \frac{\lambda_2(1-\lambda_3)}{\lambda_4(1+\lambda_4)}p^{\lambda_4} + \frac{\lambda_2\lambda_3}{\lambda_4(1+\lambda_4)p} - \frac{\lambda_2\lambda_3(1-p)^{\lambda_4+1}}{\lambda_4(1+\lambda_4)p}$ |
| van Staden-King | | | $\alpha + \beta(1-\delta)\log p - \beta\delta\log(1-p)$ | $\alpha + \beta\delta - \beta(1-\delta) + \beta(1-\delta)\log p$ $\beta\delta\frac{1-p}{p}\log(1-p)$ |
| Loglog | $\alpha\log(\lambda)x^{\alpha-1} \cdot \lambda^{x^\alpha}\exp\left[1-\lambda^{x^\alpha}\right]$, $x>0$ | $1-\exp\left[1-\lambda^{x^\alpha}\right]$ | $\left\{\frac{\log[1-\log(1-p)]}{\log\lambda}\right\}^{1/\alpha}$ | $\frac{1}{p(\log\lambda)^{1/\alpha}} \cdot \int_0^p \left\{\log\left[1 - \log(1-v)\right]\right\}^{1/\alpha} dv$ |
| Exponential logarithmic | $-\frac{\beta(1-a)\exp(-\beta x)}{\log a[1-(1-a)\exp(-\beta x)]}$, $x>0$ | $1-\frac{\log[1-(1-a)\exp(-\beta x)]}{\log a}$ | $-\frac{1}{\beta}\log\left[\frac{1-a^{1-p}}{1-a}\right]$ | $-\frac{1}{\beta p}\int_0^p \log \left[\frac{1-a^{1-v}}{1-a}\right] dv$ |
| Exponential geometric | $\frac{\lambda\theta\exp(-\lambda x)}{[1-(1-\theta)\exp(-\lambda x)]^2}$, $x>0$ | $\frac{\theta\exp(-\lambda x)}{1-(1-\theta)\exp(-\lambda x)}$ | $-\frac{1}{\lambda}\log\frac{p}{\theta+(1-\theta)p}$ | $-\frac{\log p}{\lambda} - \frac{\theta\log\theta}{\lambda p(1-\theta)} + \frac{\theta+(1-\theta)p}{\lambda p(1-\theta)} \cdot \log[\theta+(1-\theta)p]$ |

| | | | | |
|---|---|---|---|---|
| Exponential Poisson | $\frac{\beta\lambda\exp[-\beta x-\lambda+\lambda\exp(-\beta x)]}{1-\exp(-\lambda)}$, $x>0$ | $\frac{1-\exp[-\lambda+\lambda\exp(-\beta x)]}{1-\exp(-\lambda)}$ | $-\frac{1}{\beta}\log\left\{\frac{1}{\lambda}\log\left[1-p+p\exp(-\lambda)\right]+1\right\}$ | $-\frac{1}{\beta p}\int_0^p\log\left\{\frac{1}{\lambda}\log\left[1-v+v\exp(-\lambda)\right]+1\right\}dv$ |
| Topp-Leone | $2b(x(2-x))^{b-1}(1-x)$, $0\le x\le 1$ | $(x(2-x))^b$ | $1-\sqrt{1-p^{1/b}}$ | $1-\frac{b}{p}B_{p^{1/b}}\left(b,\frac{3}{2}\right)$ |
| Quadratic | $\alpha(x-\beta)^2$, $a\le x\le b$, $\alpha=\frac{12}{(b-a)^3}$, $\beta=\frac{a+b}{2}$ | $\frac{\alpha}{3}\left[(x-\beta)^3+(\beta-a)^3\right]$ | $\beta+\left[\frac{3p}{\alpha}-(\beta-a)^3\right]^{1/3}$ | $\beta+\frac{\alpha}{4p}\left\{\left[\frac{3p}{\alpha}-(\beta-a)^3\right]^{4/3}-(\beta-a)^4\right\}$ |
| Schabe | $\frac{2\gamma+(1-\gamma)x/\theta}{\theta(\gamma+x/\theta)^2}$, $x>0$ | $\frac{(1+\gamma)x}{x+\gamma\theta}$ | $\frac{p\gamma\theta}{1+\gamma-p}$ | $-\theta\gamma-\frac{\theta\gamma(1+\gamma)}{p}\cdot\log\frac{1+\gamma-p}{1+\gamma}$ |
| Birnbaum Saunders | $\frac{x^{1/2}+x^{-1/2}}{2\gamma x}\cdot\phi\left(\frac{x^{1/2}-x^{-1/2}}{\gamma}\right)$, $x>0$ | $\Phi\left(\frac{x^{1/2}-x^{-1/2}}{\gamma}\right)$ | $\frac{1}{4}\left\{\gamma\Phi^{-1}(p)+\sqrt{4+\gamma^2\left[\Phi^{-1}(p)\right]^2}\right\}^2$ | $\frac{1}{4p}\int_0^p\left\{\gamma\Phi^{-1}(v)+\sqrt{4+\gamma^2\left[\Phi^{-1}(v)\right]^2}\right\}^2dv$ |
| Generalized extreme value | $\frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi-1}\cdot\exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$, $x\ge\mu-\sigma/\xi$ if $\xi>0$, $x\le\mu-\sigma/\xi$ if $\xi<0$, $-\infty<x<\infty$ if $\xi=0$ | $\exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}$ | $\mu-\frac{\sigma}{\xi}+\frac{\sigma}{\xi}(-\log p)^{-\xi}$ | $\mu-\frac{\sigma}{\xi}+\frac{\sigma}{p\xi}\int_0^p(-\log v)^{-\xi}dv$ |