EXAMPLE CLASS

TUES 6 MARCH

15:00-16:00PM

MATH20802

Q3

$X_1, \ldots, X_n$ IID $N(\mu, 1)$

$$\hat{\mu} = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$= \frac{1.466 + 1.791 + \ldots + (-0.237)}{8}$$

$$= \ldots$$

$$P(X < 0) = P(N(\mu, 1) < 0)$$

$$= P(N(\mu, 1) - \mu < 0 - \mu)$$

$$= P(N(0, 1) < -\mu)$$

CDF of $N(0,1)$ → $= \Phi(-\mu)$

⇒ The MLE of $P(X<0)$ is

$$\Phi(-\hat{\mu}) = \Phi(-\bar{X}).$$

Q4

$X_1, \dots, X_n$ IID $N(\alpha+\beta, 1)$

$$\widehat{\alpha+\beta} = \bar{X}$$

$$\Rightarrow \hat{\alpha} + \hat{\beta} = \bar{X} \quad \text{---} \quad (1)$$

$Y_1, \dots, Y_n$ IID $N(\alpha-\beta, 1)$

$$\widehat{\alpha-\beta} = \bar{Y}$$

$$\Rightarrow \hat{\alpha} - \hat{\beta} = \bar{Y} \quad \text{---} \quad (2)$$

$(1)+(2) \Rightarrow 2\hat{\alpha} = \bar{X} + \bar{Y} \Rightarrow \hat{\alpha} = \dfrac{\bar{X}+\bar{Y}}{2}$

$(1)-(2) \Rightarrow 2\hat{\beta} = \bar{X} - \bar{Y} \Rightarrow \hat{\beta} = \dfrac{\bar{X}-\bar{Y}}{2}$

Another way:

$$L(\alpha, \beta) = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{(x_i-\alpha-\beta)^2}{2}} \right] \cdot \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{(y_i-\alpha+\beta)^2}{2}} \right]$$

Q5

$$L(\lambda) = \prod_{i=1}^{n} \left[\frac{x_i^{r-1} \lambda^r e^{-\lambda x_i}}{\Gamma(r)} \right]$$

$$= \frac{\lambda^{nr}}{(\Gamma(r))^n} \left(\prod_{i=1}^{n} x_i \right)^{r-1} e^{-\lambda \sum_{i=1}^{n} x_i}$$

$$\log L(\lambda) = nr \log \lambda - n \log \Gamma(r) + (r-1) \sum_{i=1}^{n} \log x_i - \lambda \sum_{i=1}^{n} x_i$$

$$\frac{d \log L(\lambda)}{d\lambda} = \frac{nr}{\lambda} - \sum_{i=1}^{n} x_i = 0 \Rightarrow \hat{\lambda} = \left(\frac{\bar{x}}{r} \right)^{-1}$$

$$\frac{d^2 \log L(\lambda)}{d\lambda^2} = -\frac{nr}{\lambda^2} < 0 \quad \forall \ \lambda$$

Hence $\hat{\lambda} = \left(\frac{\bar{x}}{r} \right)^{-1}$ is the MLE

$$\hat{I} = (2\hat{\lambda} - 1)^2 = \left(\frac{2}{\left(\frac{\bar{x}}{r} \right)} - 1 \right)^2.$$

Q6

$X_1, \ldots, X_n$ IID $Exp(\lambda_1)$

$$\Rightarrow \hat{\lambda_1} = \frac{1}{\bar{X}} \quad \text{---------- (1)}$$

$Y_1, \ldots, Y_m$ IID $Exp(\lambda_1 \lambda_2)$

$$\Rightarrow \widehat{\lambda_1 \lambda_2} = \frac{1}{\bar{Y}} \quad \text{---------- (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \hat{\lambda_2} = \frac{\bar{X}}{\bar{Y}}$$

$$\hat{\lambda_1} = \frac{1}{\bar{X}}$$

Another way

$$L(\lambda_1, \lambda_2) = \prod_{i=1}^{n} \left[\lambda_1 e^{-\lambda_1 x_i}\right] \cdot \prod_{i=1}^{m} \left[\lambda_1 \lambda_2 e^{-\lambda_1 \lambda_2 y_i}\right]$$