LECTURE CLASS

TUES 27 FEB

16:00-17:00PM

MATH20802

Q5

$$L(p) = \prod_{i=1}^{n} \left[p(1-p)^{x_i - 1} \right]$$

$$= p^n (1-p)^{\sum_{i=1}^{n} (x_i - 1)}$$

$$= p^n (1-p)^{\left(\sum_{i=1}^{n} x_i\right) - n}$$

$$\log L(p) = n \log p + \left[\left(\sum_{i=1}^{n} x_i\right) - n \right] \log (1-p)$$

$$\frac{d \log L(p)}{dp} = \frac{n}{p} - \frac{\left(\sum_{i=1}^{n} x_i\right) - n}{1-p} = 0$$

$$\Rightarrow \hat{p} = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}}$$

$$\frac{d^2 \log L(p)}{dp^2} = -\frac{n}{p^2} - \frac{\left(\sum_{i=1}^{n} x_i\right) - n}{(1-p)^2} < 0 \quad \forall p$$

because $\left(\sum_{i=1}^{n} x_i\right) - n \geq 0$

Proof: $x_i \geq 1 \quad \forall i$

$$\Rightarrow \sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} 1$$

$$\Rightarrow \sum_{i=1}^{n} x_i \geq n$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n \geq 0.$$

Hence, $\hat{p} = \frac{1}{\bar{x}}$ is the MLE.

Q6 $X \sim Bin(2, p)$

x	$P(X=x)$
0	$\binom{2}{0} p^0 (1-p)^2 = (1-p)^2$
1	$\binom{2}{1} p^1 (1-p)^1 = 2p(1-p)$
2	$\binom{2}{2} p^2 (1-p)^0 = p^2$

n_0 0's
n_1 1's
n_2 2's

$$L(p) = [(1-p)^2]^{n_0} [2p(1-p)]^{n_1} [p^2]^{n_2}$$
$$= 2^{n_1} p^{n_1 + 2n_2} (1-p)^{2n_0 + n_1}$$
$$\log L(p) = n_1 \log 2 + (n_1 + 2n_2) \log p + (2n_0 + n_1) \log (1-p)$$
$$\frac{d \log L}{dp} = \frac{n_1 + 2n_2}{p} - \frac{2n_0 + n_1}{1-p} = 0$$
$$\Rightarrow \hat{p} = \frac{n_1 + 2n_2}{2(n_0 + n_1 + n_2)} = \frac{n_1 + 2n_2}{2N}$$
$$\frac{d^2 \log L}{dp^2} = -\frac{n_1 + 2n_2}{p^2} - \frac{2n_0 + n_1}{(1-p)^2} < 0 \quad \forall \ p$$

Hence $\hat{p}$ is an MLE.

Toss a coin

H	$\frac{1}{2}$
HH	$\left(\frac{1}{2}\right)^2$
HHH	$\left(\frac{1}{2}\right)^3$
HHHH	$\left(\frac{1}{2}\right)^4$
⋮	

Q7

(i)

$$L(p) = \binom{70}{58} p^{58} (1-p)^{12}$$

[there is 1 observation from $Bin(70, p)$]

$$L(p) = \prod_{i=1}^{70} \left[p^{x_i} (1-p)^{1-x_i} \right]$$

[70 observations from Bernoulli(p)

$x_i = \begin{cases} 1 & \text{if bought by a man} \\ 0 & \text{" " " " woman} \end{cases}$

$$= p^{\sum_{i=1}^{70} x_i} (1-p)^{70 - \sum_{i=1}^{70} x_i}$$

$$= p^{58} (1-p)^{12}$$

(ii) $\log L(p) = 58 \log p + 12 \log(1-p)$

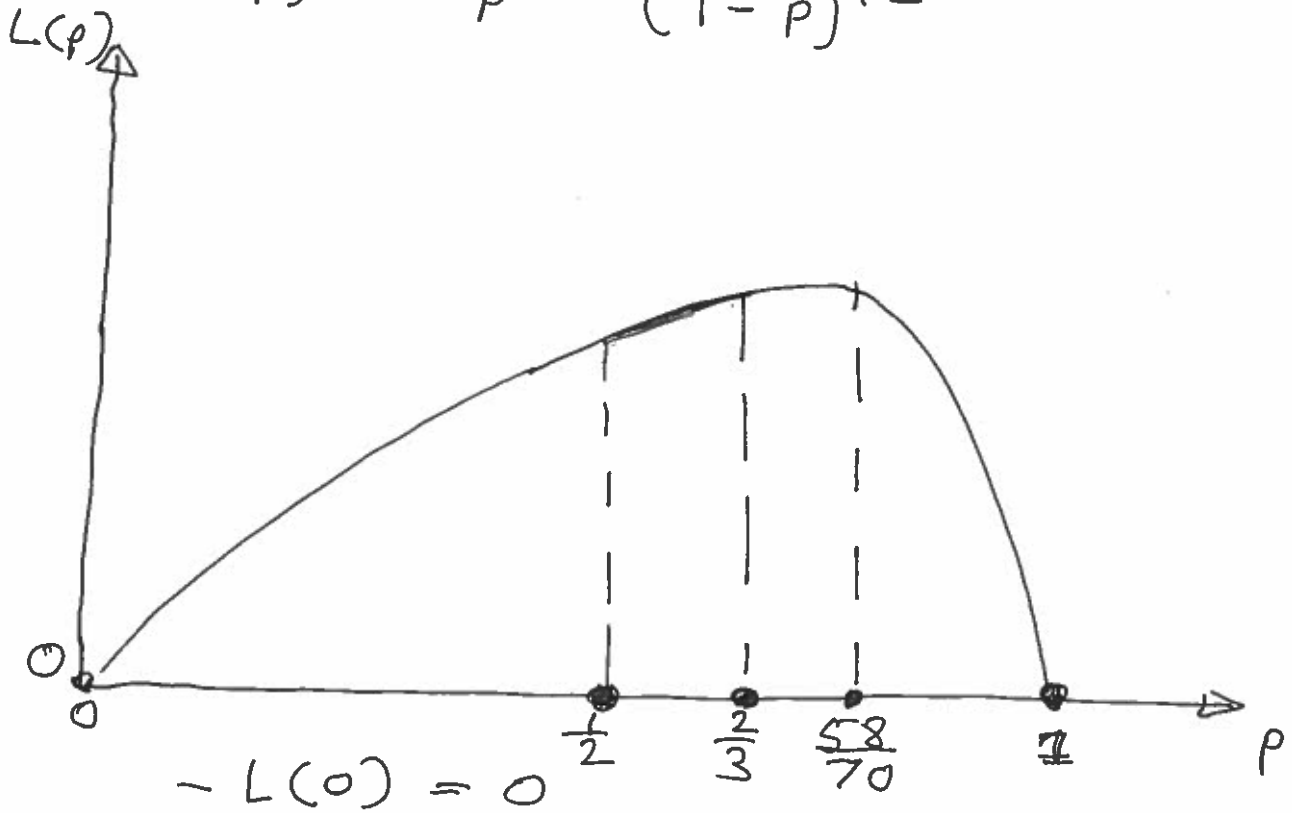
$$\frac{d \log L(p)}{dp} = \frac{58}{p} - \frac{12}{1-p} = 0$$

$$\Rightarrow \quad \hat{p} = \frac{58}{70}$$

$$\frac{d^2 \log L(p)}{dp^2} = -\frac{58}{p^2} - \frac{12}{(1-p)^2} < 0 \quad \forall p$$

$\Rightarrow \hat{p}$ is the MLE.

(iii) $L(p) = p^{58}(1-p)^{12}$



- $L(0) = 0$
- $L(1) = 0$
- $L(p)$ is maximized at $p = \frac{58}{70}$

Hence the MLE $\hat{p} = \frac{2}{3}$.