REVISION

CLASS

Q1 MGFs

Q2 Bias, MSE, consistency, unbiasedness

Q3 1- parameter MLE
You MUST SHOW $\frac{d^2 \log L}{d\theta^2} < 0$

~~Q4~~

Q4 2- parameter MLE
No need to show that Hessian has positive determinant

~~Q6~~

ANY FOUR

Q5 Hypothesis tests, power function, type I error, type II error

Q6 N-P Lemma

Ex 55

$H_0: \mu = \mu_0$

$H_1: \mu = \mu_1, \ \mu_1 > \mu_0$

Reject H_0 if $\bar{X} > k, \ \sigma^2 = 1$

$\alpha = 0.05$

$$\Rightarrow P(\bar{X} > k \mid \mu = \mu_0) = 0.05$$

$$\Rightarrow P\left(\frac{\bar{X} - \mu_0}{1/\sqrt{n}} > \frac{k - \mu_0}{1/\sqrt{n}} \,\middle|\, \mu = \mu_0\right) = 0.05$$

$$\boxed{\bar{X} \sim N\left(\mu_0, \frac{1}{n}\right)}$$

$$\Rightarrow P\left(N(0,1) > \sqrt{n}(k - \mu_0) \,\middle|\, \mu = \mu_0\right) = 0.05$$

$$\Rightarrow 1 - \Phi\left(\sqrt{n}(k - \mu_0)\right) = 0.05$$

$$\Rightarrow \Phi\left(\sqrt{n}(k - \mu_0)\right) = 0.95$$

$$\Rightarrow \sqrt{n}(k - \mu_0) = \Phi^{-1}(0.95)$$

$$\Rightarrow k = \mu_0 + \frac{1}{\sqrt{n}} \Phi^{-1}(0.95).$$

$H_0 : \mu = 10$

$H_1 : \mu = 11$

$\beta = 0.05$

$$P(\text{Accept } H_0 \mid \mu = 11) = 0.05$$

$$\Leftrightarrow P\left(\bar{X} \leq 10 + \frac{1}{\sqrt{n}}\Phi^{-1}(0.95) \,\middle|\, \mu = 11\right) = 0.05$$

$$\Leftrightarrow P\left(\frac{\bar{X} - 11}{\frac{1}{\sqrt{n}}} \leq \frac{10 + \frac{1}{\sqrt{n}}\Phi^{-1}(0.95) - 11}{\frac{1}{\sqrt{n}}} \,\middle|\, \mu = 11\right) = \overset{0.05}{\cancel{0.95}}$$

$$\Leftrightarrow \Phi\left(10\sqrt{n} + \Phi^{-1}(0.95) - 11\sqrt{n}\right) = \overset{0.05}{\cancel{0.95}}$$

$$\Leftrightarrow \Phi\left(\Phi^{-1}(0.95) - \sqrt{n}\right) = \overset{0.05}{\cancel{0.95}}$$

$$\Leftrightarrow \Phi^{-1}(0.95) - \sqrt{n} = \Phi^{-1}(0.05)$$

$$\Leftrightarrow \sqrt{n} = \Phi^{-1}(0.95) - \Phi^{-1}(0.05)$$

$$\Leftrightarrow n = \left[\Phi^{-1}(0.95) - \Phi^{-1}(0.05)\right]^2.$$

Ex 48

$$f(x) = \begin{cases} \frac{1}{4} & x = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

$n = 36$

$$P(1.4 < \bar{X} < 1.8)$$

$$= P\left(\frac{1.4 - \mu}{\sigma/\sqrt{36}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{36}} < \frac{1.8 - \mu}{\sigma/\sqrt{36}} \right)$$

$$\underset{CLT}{\overset{by}{\approx}} P\left(\frac{6(1.4-\mu)}{\sigma} < N(0,1) < \frac{6(1.8-\mu)}{\sigma} \right)$$

$$= \Phi\left(\frac{6 \cdot (1.8-\mu)}{\sigma} \right) - \Phi\left(\frac{6 \cdot (1.4-\mu)}{\sigma} \right)$$

$$\mu = E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4}$$

$$= 1.5$$

$$\sigma^2 = Var(X) = E(X^2) - (1.5)^2$$

$$= 0^2 \cdot \frac{1}{4} + 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{4} - (1.5)^2$$

$$= \frac{7}{2} - (1.5)^2.$$