REVISION CLASS

THURS 3 MAY

16:00-17:00PM

MATH20802

Ex 87 $X \sim B(a, 1)$. Find the most powerful test of $H_0: a = 1$ vs $H_1: a = 2$

$\alpha = 0.05$

By N-P Lemma, we reject H_0 if

$$\frac{L(1)}{L(2)} < k$$

$$\Leftrightarrow \frac{\frac{\Gamma(1+1)}{\Gamma(1)\Gamma(1)} x^{1-1}(1-x)^{1-1}}{\frac{\Gamma(2+1)}{\Gamma(2)\Gamma(1)} x^{2-1}(1-x)^{1-1}} < k$$

$$\Leftrightarrow \frac{1}{2x} < k$$

$$\Leftrightarrow \boxed{x > \frac{1}{2k}}$$ What is k?

$\alpha = 0.05$

$$\Leftrightarrow P\left(X > \frac{1}{2k} \mid a = 1\right) = 0.05$$

$$\Leftrightarrow P\left(B(a,1) > \frac{1}{2k} \mid a = 1\right) = 0.05$$

$$\Leftrightarrow P\left(B(1,1) > \frac{1}{2k}\right) = 0.05$$

$$\Leftrightarrow P\left(\text{Uni}[0,1] > \frac{1}{2k}\right) = 0.05$$

$$\Leftrightarrow P\left(\text{Uni}[0,1] \leq \frac{1}{2k}\right) = 0.95$$

$$\Leftrightarrow \frac{1}{2k} = 0.95$$

$P(\text{Uni}[0,1] \leq x) = x$

We reject H_0 at $\alpha = 0.05$ if $X > 0.95$

Ex 88 $X_1, X_2, \ldots, X_{20}$ IID Bernoulli(p).

Find the most powerful test of

$H_0 : p = 0.2$

vs $H_1 : p = 0.3$

$\alpha = 0.05$

By N-P Lemma, we reject H_0 if

$$\frac{L(0.2)}{L(0.3)} < k$$

$$\Leftrightarrow \frac{\prod_{i=1}^{20} \left[(0.2)^{x_i} (1-0.2)^{1-x_i} \right]}{\prod_{i=1}^{20} \left[(0.3)^{x_i} (1-0.3)^{1-x_i} \right]} < k$$

$$\Leftrightarrow \frac{(0.2)^{\sum_{i=1}^{20} x_i} (0.8)^{20 - \sum_{i=1}^{20} x_i}}{(0.3)^{\sum_{i=1}^{20} x_i} (0.7)^{20 - \sum_{i=1}^{20} x_i}} < k$$

$$\Leftrightarrow \left(\frac{7}{12}\right)^{\sum_{i=1}^{20} x_i} \left(\frac{8}{7}\right)^{20} < k$$

$$\Leftrightarrow \left(\frac{7}{12}\right)^{\sum_{i=1}^{20} x_i} < \left(\frac{7}{8}\right)^{20} k$$

$$\Leftrightarrow \left(\sum_{i=1}^{20} x_i\right) \boxed{\log\left(\frac{7}{12}\right)} < \log\left[\left(\frac{7}{8}\right)^{20} k\right]$$

$$\Leftrightarrow \sum_{i=1}^{20} x_i > \frac{\log\left[\left(\frac{7}{8}\right)^{20} k\right]}{\log\left(\frac{7}{12}\right)} = c$$

What is $<$?

$\alpha = 0.05$

$$\Rightarrow P\left(\sum_{i=1}^{20} X_i > c \mid p = 0.2\right) = 0.05$$

$$\Rightarrow P\left(\text{Bin}(20, 0.2) > c\right) = 0.05$$

$$\Rightarrow P\left(\text{Bin}(20, 0.2) \leq c\right) = 0.95$$

$$\Rightarrow \boxed{\sum_{i=0}^{c} \binom{20}{i} (0.2)^i (1-0.2)^{20-i} = 0.95} \quad (*)$$

This must be solved numerically for c.

We reject H_0 at $\alpha = 0.05$ if $\sum_{i=1}^{20} X_i > c$, where c satisfies $(*)$.