

**MATH20802: STATISTICAL METHODS**  
**SEMESTER 2**  
**SOLUTION TO QUIZ PROBLEM 9**

Suppose  $X$  is a  $\text{beta}(a, 1)$  random variable. Consider testing  $H_0 : a = a_0$  versus  $H_1 : a = a_1$ . Choose to reject  $H_0$  if  $X > 1/2$ . The power function of  $a$  is

$$\begin{aligned}\Pi(a) &= \Pr(\text{Reject } H_0 \mid a) \\ &= \Pr(X > 1/2 \mid a) \\ &= \int_{1/2}^1 \frac{\Gamma(a+1)}{\Gamma(a)\Gamma(1)} x^{a-1} (1-x)^{1-1} dx \\ &= a \int_{1/2}^1 x^{a-1} dx \\ &= [x^a]_{1/2}^1 \\ &= 1 - (1/2)^a.\end{aligned}$$