MATH20802: STATISTICAL METHODS
SEMESTER 2
SOLUTION TO QUIZ PROBLEM 8

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution specified by the probability density function

$$f(x) = \frac{1}{2a} \exp\left(-\frac{\mid x - b \mid}{a}\right)$$

for $-\infty < x < \infty$, where both $-\infty < b < \infty$ and $a > 0$ are unknown parameters. The joint likelihood function of a and b is

$$L(a, b) = \prod_{i=1}^{n} \left[\frac{1}{2a} \exp\left(-\frac{\mid X_i - b \mid}{a}\right)\right] = \frac{1}{(2a)^n} \exp\left(-\frac{1}{a} \sum_{i=1}^{n} \mid X_i - b \mid\right).$$

The log likelihood function is

$$\log L(a, b) = -n \log(2a) - \frac{1}{a} \sum_{i=1}^{n} \mid X_i - b \mid .$$

The partial derivatives with respect to a and b are

$$\frac{\partial \log L(a, b)}{\partial a} = -\frac{n}{a} + \frac{1}{a^2} \sum_{i=1}^{n} \mid X_i - b \mid \tag{1}$$

and

$$\frac{\partial \log L(a, b)}{\partial b} = -\frac{1}{a^2} \sum_{i=1}^{n} \mathrm{sign}\,(X_i - b)\,. \tag{2}$$

Setting (2) to zero gives $\widehat{b} = \mathrm{Median}\,(X_1, \ldots, X_n)$. From (1), we obtain

$$\widehat{a} = \frac{1}{n} \sum_{i=1}^{n} \mid X_i - \mathrm{Median}\,(X_1, \ldots, X_n) \mid .$$