MATH20802: STATISTICAL METHODS
SEMESTER 2
SOLUTION TO QUIZ PROBLEM 2

Suppose X is a random variable with probability density function

$$f(x) = \frac{1}{x^3} \exp\left(-\frac{1}{x}\right)$$

for $x > 0$. Using the fact that $\exp(z) > z^2/2!$ for $z > 0$, we can write

$$\begin{aligned}
M_X(t) &= E\left[\exp(tX)\right] \\
&= \int_0^\infty \exp(tx) \frac{1}{x^3} \exp\left(-\frac{1}{x}\right) dx \\
&> \int_0^\infty \frac{(tx)^2}{2} \frac{1}{x^3} \exp\left(-\frac{1}{x}\right) dx \\
&= \frac{t^2}{2} \int_0^\infty \frac{1}{x} \exp\left(-\frac{1}{x}\right) dx.
\end{aligned}$$

Setting $y = 1/x$, we obtain

$$\begin{aligned}
M_X(t) &> \frac{t^2}{2} \int_\infty^0 y \left[\exp\left(-y\right)\right] \left(-\frac{1}{y^2}\right) dy \\
&= \frac{t^2}{2} \int_0^\infty \frac{1}{y} \exp\left(-y\right) dy \\
&= \frac{t^2}{2} \Gamma(0) \\
&= +\infty.
\end{aligned}$$

Hence, $M_X(t)$ does not exist for $t > 0$.