

**MATH20802: STATISTICAL METHODS**  
**SEMESTER 2**  
**SOLUTION TO QUIZ PROBLEM 10**

Suppose  $X_1, X_2, \dots, X_n$  is a random sample from the  $\text{Exp}(a)$  distribution. Consider testing  $H_0 : a = a_0$  versus  $H_1 : a = a_1$ . Choose to reject  $H_0$  if  $\min(X_1, X_2, \dots, X_n) > 1$ . The power function of  $a$  is

$$\begin{aligned}\Pi(a) &= \Pr(\text{Reject } H_0 \mid a) \\ &= \Pr(\min(X_1, X_2, \dots, X_n) > 1 \mid a) \\ &= \Pr(X_1 > 1, X_2 > 1, \dots, X_n > 1 \mid a) \\ &= \Pr(X_1 > 1 \mid a) \Pr(X_2 > 1 \mid a) \cdots \Pr(X_n > 1 \mid a) \\ &= [1 - \Pr(X_1 \leq 1 \mid a)] [1 - \Pr(X_2 \leq 1 \mid a)] \cdots [1 - \Pr(X_n \leq 1 \mid a)] \\ &= [1 - (1 - e^{-a})] [1 - (1 - e^{-a})] \cdots [1 - (1 - e^{-a})] \\ &= e^{-a} e^{-a} \cdots e^{-a} \\ &= e^{-na}.\end{aligned}$$