MATH20802: STATISTICAL METHODS
SEMESTER 2
SOLUTION TO QUIZ PROBLEM 1

Suppose X is a random variable with probability mass function

$$p(k) = \Pr(X = k) = -\frac{p^k}{k \log(1-p)}$$

for $0 < p < 1$ and $k = 1, 2, \ldots$. Using the Taylor expansion

$$\log(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k},$$

the moment generating function of X can be expressed as

$$\begin{aligned} M_X(t) &= -\sum_{k=1}^{\infty} e^{tk} \Pr(X = k) \\ &= -\sum_{k=1}^{\infty} e^{tk} \frac{p^k}{k \log(1-p)} \\ &= -\frac{1}{\log(1-p)} \sum_{k=1}^{\infty} \frac{\left[e^t p\right]^k}{k} \\ &= \frac{1}{\log(1-p)} \log\left[1 - e^t p\right]. \end{aligned}$$