MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 8

1. Let X_1, X_2, X_3 be a random sample from the $Po(\lambda)$ distribution.

   (i) Find the most powerful test at level α for $H_0 : \lambda = 1$ versus $\lambda = 2$.

   (ii) Show the this test is equivalent to rejecting H_0 if and only if $T = X_1 + X_2 + X_3 \geq k$ for some k.

   (iii) Find the value of k when $\alpha = 0.1$. You may assume that T has a $Po(3\lambda)$ distribution.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(0, \sigma^2)$ distribution.

   (i) Find the most powerful test at level α for $H_0 : \sigma = \sigma_0$ versus $\sigma = \sigma_1$, where $\sigma_0 < \sigma_1$ are constants. Show that the test rejects H_0 if and only if $\sum_{i=1}^{n}(X_i/\sigma_0)^2 > k$ for some k.

   (ii) Find the value of k when $\alpha = 0.05$.

   (iii) Find $\beta = \Pr$ (Type II error) when $n = 10$, $\sigma_0 = 2$ and $\sigma_1 = 3$ in terms of the cdf of a χ^2 random variable.