MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 7

1. Verify the identity

$$\sum_{i=1}^{n}(X_i-\mu)^2=\sum_{i=1}^{n}(X_i-\bar{X})^2+n(\bar{X}-\mu)^2.$$

2. If $Z\sim N(0,1)$ show that $Z^2\sim\chi^2_1$.

3. If $Z\sim N(0,1)$ and χ^2_ν are independent random variables show that $T=Z/\sqrt{\chi^2_\nu/\nu}\sim t_\nu$.

4. A random sample of 20 observations are taken from $N(\mu,1.4)$ distribution when μ is unknown. Find two numbers a and b such that $\Pr(a\leq S^2\leq b)=0.95$, where S^2 denotes the sample variance.

5. In 16 test runs the petrol consumption of an experimental engine has a standard deviation of 2.2 gallons. Stating any distributional assumptions you make, test whether σ, the true standard deviation of the petrol consumption of the engine, is equal to 4.5 gallons.

6. If $X\sim F_{\nu_1,\nu_2}$ explain why $1/X\sim F_{\nu_2,\nu_1}$. Show that $F_{\nu_1,\nu_2,1-\alpha}=1/F_{\nu_2,\nu_1,\alpha}$.