MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 6

1. The following are the values of independent samples from two different populations.

   Sample X: 122, 114, 130, 165, 144, 133, 139, 142, 150.
   Sample Y: 108, 125, 122, 140, 132, 120, 137, 128, 138.

   Let μ_X and μ_Y be the respective means of the two populations. Find the p-value of the test of the null hypothesis $H_0 : \mu_X \leq \mu_Y$ against the alternative $H_1 : \mu_X > \mu_Y$ when the population standard deviations are $\sigma_X = 10$ and: a) $\sigma_Y = 5$, b) $\sigma_Y = 10$, c) $\sigma_Y = 20$.

2. The value received at a certain message receiving station is equal to the value sent plus a random error that is normal with mean 0 and standard deviation 2. Two messages, each consisting of a single value, are to be sent. Because of the random error, each message will be sent 9 times. Before reception, the receiver is fairly certain that the first message value will be less than or equal to the second. Should this hypothesis be rejected if the average of the values relating to message 1 is 5.6 whereas the average of those relating to message 2 is 4.1? Use the 1 percent level of significance.

3. Data were collected to determine if there is a difference between the mean IQ scores of urban and rural students in upper Michigan. A random sample of 100 urban students yielded a sample mean score of 102.2 and a sample standard deviation of 11.8. A random sample of 60 rural students yielded a sample mean score of 105.3 with a sample standard deviation of 10.6. Are the data significant enough, at the 5 percent level, for us to reject the hypothesis that the mean scores of urban and rural students are the same? Perform the test for the case where the variances are equal and for the case where they are not equal.

4. In the above problem, are the data significant enough, at the 1 percent level, to conclude that the mean score of rural students in upper Michigan is greater than that of urban students?

5. To see whether there are any differences in starting salaries for women and men law school graduates, a set of eight law firms was selected. For each of these firms a recently hired woman and a recently hired man were randomly chosen. The following starting salary information resulted from interviewing those chosen.

Company	1	2	3	4	5	6	7	8
Woman's salary	52	53.2	78	75	62.5	72	39	49
Man's salary	54	55.5	78	81	64.5	70	42	51

   Use the above data to test the hypothesis, at the 10 percent level of significance, that the starting salary is the same for both sexes.

6. The following data give the marriage rates per 1000 population in a random sample of countries for 1987 and 1989.

Country	1987 Rate	1989 Rate
Belgium	5.8	6.4
Finland	5.4	5.1
Greece	6.3	6.0
Israel	6.9	7.0
New Zealand	6.0	6.1
Norway	5.0	4.9
Switzerland	6.6	6.8
United States	9.9	9.7
Yugoslavia	7.0	6.7

Test the hypothesis that the worldwide marriage rates in 1987 are no greater than those in 1989. Use the 5 percent level of significance.

7. The American Cancer Society recently sampled 2500 adults and determined that 738 of them were smokers. A similar poll of 2000 adults carried out in 1986 yielded a total of 640 smokers. Do these figures prove that the proportion of adults who smoke has decreased since 1986? Use the 5 percent level of significance.

8. Suppose a random sample of 480 heart-bypass operations at hospital A showed that 72 patients did not survive, whereas a random sample of 360 operations at hospital B showed that 30 patients did not survive. Find the p-value of the test of the hypothesis that the survival probabilities are the same at the two hospitals.

9. Life threatening heart arrhythmias can be predicted from an electrocardiogram by measuring the lengths of QT intervals (the distances from the starts of the Q-waves to the starts of the T-waves). Two different calipers, A and B, were used to measure a set of 10 QT intervals. The sample variances for the two calipers were 831 and 592, respectively. Do these data suggest that the calipers A and B have different variability? Use the 5 percent level of significance.

10. A restaurant adds a new commercial oven to its kitchen. It is hoped that the new oven has more evenly distributed heat than the current oven. The ovens are heated to 350F, using a thermostat control, and temperature readings are obtained from thermometers placed at 9 locations in each oven, yielding the following data:

    Current oven: $m = 9$, $\bar{x} = 352.4$ and $s_X = 2.3$
    New oven: $n = 9$, $\bar{y} = 350.2$ and $s_Y = 1.1$

    Test $H_0 : \sigma_X^2 = \sigma_Y^2$ versus $H_1 : \sigma_X^2 > \sigma_Y^2$ using the 5 percent level of significance.