MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 5

1. The proportion p of defective items in a large population is unknown. We wish to test $H_0 : p = 0.2$ versus $H_1 : p \neq 0.2$. Suppose a random sample of 20 items is drawn from the population. Let Y be the number of defective items in the sample. Consider the test which rejects H_0 if and only if $Y \geq 7$ or $Y \leq 1$. Find the power function $\Pi(p)$ of this test when $p = 0, 0.2, 0.4, 0.6, 0.8, 1$. What is the level of significance of the test?

2. In a sequence of Bernoulli trials the probability p of success at each trial is unknown. Let X be the number of trials up to and including the first success (X has the geometric distribution). We wish to test $H_0 : p = 0.1$ versus $H_1 : p = 0.2$. Consider the test procedure that rejects H_0 if and only if $X \leq 5$. Find the probabilities of a type I error and a type II error of this test.

3. Suppose we wish to test $H_0 : \mu = 0$ versus $H_1 : \mu = 4$ on the basis of a random sample $X_1, X_2, \ldots, X_n \sim N(\mu, 4)$ at the level $\alpha = 0.05$. Find an expression for the probability of a type II error using the usual test procedure.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $N(\mu, 4)$ distribution. Show that the power function for a test of $H_0 : \mu = 0$ versus $H_1 : \mu \neq 0$ at level α is $\pi\left(\mu^{'}\right) = \Phi(-z_{\alpha/2} - \mu^{'}\sqrt{n}/2) + 1 - \Phi(z_{\alpha/2} - \mu^{'}\sqrt{n}/2)$, where $\Phi(\cdot)$ is the cdf of the standard normal distribution.

5. Suppose we wish to test $H_0 : \mu = 100$ versus $H_1 : \mu < 100$ on the basis of a random sample $X_1, X_2, \ldots, X_n \sim N(\mu, 20)$ at the level $\alpha = 0.05$. If we require that with probability at least 0.9 the standard test procedure will reject H_0 when in fact the true value of μ is 95, how large should n be?

6. A machine manufactures bolts whose diameters have the $N(\mu, \sigma^2)$ distribution. Write down the standard procedure for testing $H_0 : \sigma = 10^{-2}$cm versus $H_1 : \sigma > 10^{-2}$cm at the level $\alpha = 0.05$ based on a random sample of size $n = 35$. If in fact $\sigma = 2 \times 10^{-2}$cm express the probability of a type II error in terms of the cdf of a χ^2 random variable.