

MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 3

1. If the random variable $X \sim \text{Bin}(n, p)$ show that the sample proportion $\hat{p} = X/n$ is an unbiased estimator of p . Calculate $\text{Var}(\hat{p})$ and hence the MSE (\hat{p}). Is \hat{p} MSE consistent for p ?
2. Let X_1, X_2, \dots, X_n be a random sample from a $N(0, \sigma^2)$ distribution. Show that $\sum_{i=1}^n X_i^2/n$ is an unbiased estimator of σ^2 . Show that it is also consistent.
3. Let X_1, X_2, \dots, X_n be a random sample from the distribution

$$f(x) = \begin{cases} \exp(\delta - x), & \text{if } x \geq \delta, \\ 0, & \text{otherwise,} \end{cases}$$

where δ is an unknown parameter. Show that \bar{X} is a biased estimator of δ . Hence, find a linear transformation of \bar{X} which will provide an unbiased estimator of δ . Is your new estimator MSE consistent for δ ?

4. When have seen in the notes that the maximum likelihood estimator of the parameter λ in a Poisson distribution is $\hat{\lambda} = \bar{X}$. Find the expected value and the variance of $\hat{\lambda}$ and show that it is a consistent estimator for λ .
5. If X_1, X_2, \dots, X_n is a random sample from the geometric distribution (with parameter p) find the mle of p .
6. Among N independent random variable having the $\text{Bin}(2, p)$ distribution, n_0 take on the value 0, n_1 take on the value 1 while n_2 take on the value 2. We have $n_0 + n_1 + n_2 = N$. Find the mle of p .
7. The proportion p of the breakfast cereal Cocobix bought by men rather than women is unknown. In a random sample of 70 purchases of the cereal it was found that 58 were made by men.
 - (i) Write down the likelihood function $L(p)$.
 - (ii) Find the mle of p and an approximate 95% confidence interval for its true value.
 - (iii) Sketch $L(p)$ for $0 \leq p \leq 1$ and find the mle of p if we know its true values lies in the interval $1/2 \leq p \leq 2/3$.