MATH20802: STATISTICAL METHODS
SEMESTER 2
PROBLEM SHEET 0

1. IF $X \sim N(\mu, \sigma^2)$ show that its mgf is

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right).$$

2. If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ are independent then show that $aX_1 + bX_2 + c \sim N(a\mu_1 + b\mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$.

3. If $X_i \sim N(\mu, \sigma^2)$, $i = 1, 2, \ldots, n$ are iid then show that $\bar{X} \sim N(\mu, \sigma^2/n)$, where

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

   is the sample mean.

4. If $X \sim Uni(a, b)$ then show that its mgf is $M_X(t) = \{\exp(bt) - \exp(at)\}/((b-a)t)$.

5. If $X \sim Exp(\lambda)$ then show that its mgf is:

$$M_X(t) = \frac{\lambda}{\lambda - t}.$$

6. If $X \sim Ga(a, \lambda)$ then show that its mgf is:

$$M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^a.$$