

**MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 26 APRIL 2017**

YOUR FULL NAME:

YOUR ID:

This test contains two questions. Please answer BOTH the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be marked by myself. Each part of the two questions will be marked as follows: i) 100 percent of the mark if the answer and the approach leading to it are correct, significant details of the approach must be given; ii) 50 percent of the mark if the answer is not correct (or if no answer is given), but approach taken is correct, significant details of the approach must be given; iii) 0 marks if the answer is correct/incorrect but little or no details are given on the approach taken.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

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| Q1 | Q2 | Total |
|----|----|-------|
| | | |

QUESTION 1 A random variable X is said to have the Gumbel distribution, written $X \sim \text{Gumbel}(\beta)$, if its probability density function is given by

$$f_X(x) = \frac{1}{\beta} \exp\left(-\frac{x}{\beta}\right) \exp\left\{-\exp\left(-\frac{x}{\beta}\right)\right\}$$

for $-\infty < x < +\infty$ and $\beta > 0$.

(i) Show that the cumulative distribution function of X is

$$F_X(x) = \exp\left\{-\exp\left(-\frac{x}{\beta}\right)\right\}$$

for $-\infty < x < +\infty$ and $\beta > 0$. (4 marks)

(ii) Show that the moment generating function of X is

$$M_X(t) = \Gamma(1 - \beta t)$$

for $t < 1/\beta$, where $\Gamma(\cdot)$ denotes the gamma function. (4 marks)

(iii) Show that

$$E(X) = -\beta\Gamma'(1),$$

where $\Gamma'(\cdot)$ denotes the first derivative of $\Gamma(\cdot)$. (2 marks)

[Total: 10 marks]

QUESTION 2 Suppose X_1, X_2, \dots, X_n are independent and identically distributed random variables with the common probability mass function (pmf):

$$p(x) = \theta(1 - \theta)^{x-1}$$

for $x = 1, 2, \dots$ and $0 < \theta < 1$.

- (i) Write down the likelihood function of θ . (2 marks)
- (ii) Find the maximum likelihood estimator (mle) of θ . (2 marks)
- (iii) Find the mle of $\psi = 1/\theta$. (2 marks)
- (iv) Determine the bias, variance and the mean squared error of the mle of ψ . (2 marks)
- (v) Is the mle of ψ unbiased? Is it consistent? (2 marks)

[Total: 10 marks]

