MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 13 APRIL 2016

YOUR FULL NAME:

YOUR ID:

This test contains two questions. Please answer BOTH the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

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Q1	Q2	Total

QUESTION 1 Let X denote an exponential random variable with its probability density function given by

$$f_X(x) = \exp(-x)$$

for $x > 0$.

(i) Show that the moment generating function of X is

$$M_X(t) = E\left[\exp(tX)\right] = \frac{1}{1-t}$$

for $t < 1$. (2 marks)

(ii) Use your result in (i) to derive the first four moments of X. (2 marks)

(iii) If X_i are independent and identical random variables distributed as X derive the moment generating function of $Y = X_1 + \cdots + X_n$. (2 marks)

(iv) Derive the mean and variance of Y. (2 marks)

(v) What is the distribution of Y? State the name of the distribution and its parameter values. (2 marks)

[Total: 10 marks]

QUESTION 2 An electrical circuit consists of two batteries connected in series to a lightbulb. We model the battery lifetimes X_1, X_2 as independent and identically distributed $Uni(0, \theta)$ random variables. Our experiment to measure the operating time of the circuit is stopped when any one of the batteries fails. Hence, the only random variable we observe is $Y = \min(X_1, X_2)$.

(i) Show that the cumulative distribution function of the random variable Y is

$$F_Y(y) = 1 - \left(1 - \frac{y}{\theta}\right)^2$$

for $y < \theta$. (2 marks)

(ii) Write down the likelihood function of θ based on a single observation of Y. (2 marks)

(iii) Derive the maximum likelihood estimator of θ. (2 marks)

(iv) Find the bias of the estimator in part (iii). Is the estimator unbiased? (2 marks)

(v) Find the mean squared error of the estimator in part (iii). (2 marks)

[Total: 10 marks]