MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 15 APRIL 2015

YOUR FULL NAME:

YOUR ID:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

QUESTION 1 Let X be a normal random variable with mean μ and standard deviation σ. Let $Y = \exp(2X)$; that is, Y is a log-normal random variable with parameters 2μ and 2σ.

(i) Show that the moment generating function of X is

$$M_X(t) = E\left[\exp(tX)\right] = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right).$$

(2 marks)

(ii) Show that $E(Y) = M_X(2)$ and $E\left(Y^2\right) = M_X(4)$. (2 marks)

(iii) Use (i) to evaluate $E(Y)$ and $Var(Y)$. (2 marks)

(iv) Let X_i, $i = 1, 2$ be independent normal random variables with mean μ_i, $i = 1, 2$ and standard deviations σ_i, $i = 1, 2$. Use (i) to find the moment generating function of $X_1 + X_2$. (2 marks)

(v) Use (iv) to determine the distribution of $\exp\left(2X_1 + 2X_2\right)$. (2 marks)

QUESTION 2 Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $N\left(c\mu, c\sigma^2\right)$, where both μ and σ^2 are unknown parameters and c is a fixed known constant.

(i) Write down the joint likelihood function of μ and σ^2. (2 marks)

(ii) Show that the maximum likelihood estimator (mle) of μ is $\widehat{\mu} = \overline{X}/c$, where $\overline{X} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$ is the sample mean. (2 marks)

(iii) Show that the mle of σ^2 is $\widehat{\sigma^2} = \dfrac{1}{nc}\displaystyle\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$. (2 marks)

(iv) Show that the mle, $\widehat{\mu}$, is an unbiased and consistent estimator for μ. (2 marks)

(v) Show that the mle, $\widehat{\sigma^2}$, is a biased and consistent estimator for σ^2. (2 marks)