

**MATH20802: STATISTICAL METHODS  
SECOND SEMESTER  
IN CLASS TEST - 17 APRIL 2013**

**YOUR FULL NAME:**

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20 percent of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

**PLEASE DO NOT TURN OVER UNTIL I SAY SO**

**QUESTION 1** Let  $X$  be a normal random variable with mean  $\mu$  and standard deviation  $\sigma$ . Let  $Y = \exp(X)$ ; that is,  $Y$  is a log-normal random variable with parameters  $\mu$  and  $\sigma$ .

(i) Evaluate  $M_X(t) = E[\exp(tX)]$ , the moment generating function of  $X$ . (2 marks)

(ii) Show that  $E(Y) = M_X(1)$  and  $E(Y^2) = M_X(2)$ . (2 marks)

(iii) Use (i) to evaluate  $E(Y)$  and  $Var(Y)$ . (2 marks)

(iv) Let  $X_i, i = 1, 2$  be independent normal random variables with mean  $\mu_i, i = 1, 2$  and standard deviations  $\sigma_i, i = 1, 2$ . Use (i) to find the moment generating function of  $X_1 + X_2$ . (2 marks)

(v) Use (iv) to determine the distribution of  $\exp(X_1 + X_2)$ . (2 marks)





**QUESTION 2** Consider the simple linear regression model with an intercept of zero:

$$Y_i = \beta X_i + e_i, \quad i = 1, 2, \dots, n,$$

where  $e_i$  are independent and identical normal random variables with zero mean and variance  $\sigma^2$  assumed known. Moreover, suppose  $X_1, X_2, \dots, X_n$  are known constants.

- (i) Write down the likelihood function of  $\beta$ . (2 marks)
- (ii) Derive the maximum likelihood estimator  $\hat{\beta}$ . (2 marks)
- (iii) Show that  $\hat{\beta}$  is unbiased for  $\beta$ . (2 marks)
- (iv) Evaluate the variance of  $\hat{\beta}$ . (2 marks)
- (iv) What is the distribution of  $\hat{\beta}$ . (2 marks)



