MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 18 APRIL 2012

YOUR FULL NAME:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20% of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

QUESTION 1 This question has four parts.

(i) Let X be a random variable with its pdf specified by

$$f(x) = \begin{cases} \theta^3, & \text{if } x = 1, \\ \theta^2(1-\theta), & \text{if } x = 2, \\ 2\theta(1-\theta), & \text{if } x = 3, \\ \theta(1-\theta)^2, & \text{if } x = 4, \\ (1-\theta)^3, & \text{if } x = 5, \\ 0, & \text{elsewhere.} \end{cases}$$

Fine the values of c for which

$$T(X) = \begin{cases} 1, & \text{if } X = 1, \\ 2-2c, & \text{if } X = 2, \\ c, & \text{if } X = 3, \\ 1-2c, & \text{if } X = 4, \\ 0, & \text{elsewhere} \end{cases}$$

is an unbiased estimator of θ. (3 marks)

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population with known variance σ^2. Find the bias and mean squared error of $S^2 = (1/n)\sum_{i=1}^{n}(X_i - \overline{X})^2$ as an estimator of σ^2, where $\overline{X} = (1/n)\sum_{i=1}^{n} X_i$. Is S^2 unbiased? Is it consistent? (2 marks)

(iii) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population with mean μ and known variance σ^2. Find the bias and mean squared error of $(X_1 + \overline{X})/2$ as an estimator of μ, where $\overline{X} = (1/n)\sum_{i=1}^{n} X_i$. Is $(X_1 + \overline{X})/2$ unbiased? Is it consistent? (3 marks)

(iv) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Find the bias of $\overline{X}^2$ as an estimator of λ^2, where $\overline{X} = (1/n)\sum_{i=1}^{n} X_i$. Is $\overline{X}^2$ unbiased? (2 marks)

QUESTION 2 This question has four parts.

(i) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x) = 2\alpha^{-2} x \exp\{-x^2/\alpha^2\}$. Find the maximum likelihood estimate of α. (2 marks)

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population with mean θ and standard deviation θ. Find the maximum likelihood estimate of θ. (3 marks)

(iii) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x) = \lambda^{-2} x \exp\{-x/\lambda\}$. Find the maximum likelihood estimate of λ. (2 marks)

(iv) Let X_1, X_2, X_3 be a random sample from the discrete uniform distribution between 1 and N, inclusive. Find the maximum likelihood estimate of N. (3 marks)