

**MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 4 MAY 2011**

YOUR FULL NAME:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20% of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

1. For a random variable X let $M_X(t) = E[\exp(tX)]$, $t > 0$ denote its moment generating function (mgf). Derive the mgfs of the following random variables:

(i) $X \sim Geom(\theta)$ with the pmf $p(x) = \theta(1 - \theta)^{x-1}$ for $x = 1, 2, \dots$ (2 marks)

(ii) $X \sim Exp(\lambda)$ with the pdf $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$. (2 marks)

(iii) a continuous random variable X with the pdf $f(x) = \exp(-x)/\{1 + \exp(-x)\}^2$ for $-\infty < x < \infty$. (2 marks)

(iv) a continuous random variable X with the pdf

$$f(x) = \begin{cases} 4x, & \text{if } 0 < x < 1/2, \\ 4(1 - x), & \text{if } 1/2 \leq x < 1 \end{cases}$$

for $0 < x < 1$. (2 marks)

(v) a continuous random variable X with the pdf $f(x) = (1/x^2) \exp(-1/x)$ for $x > 0$. (2 marks)

2. Suppose X_1, X_2, \dots, X_n are independent and identically distributed random variables with the common probability density function (pdf):

$$f(x) = \theta_2 x^{\theta_2 - 1} \theta_1^{-\theta_2}$$

for $0 < x < \theta_1$, $\theta_1 > 0$ and $\theta_2 > 0$. Both θ_1 and θ_2 are unknown.

- (i) Calculate the cumulative distribution function, mean and variance corresponding to the given pdf. (2 marks)
- (ii) Write down the joint likelihood function of θ_1 and θ_2 . (2 marks)
- (iii) Determine the maximum likelihood estimator (MLE) of θ_1 . (2 marks)
- (iv) Determine the MLE of θ_2 . (2 marks)
- (v) Show that the MLE, $\hat{\theta}_1$, is a biased and consistent estimator for θ_1 . (2 marks)

