MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 21 APRIL 2010

YOUR FULL NAME:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20% of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

PLEASE DO NOT TURN OVER UNTIL I SAY SO

1. For a random variable X let $M_X(t) = E[\exp(tX)]$ denote its moment generating function (mgf). Derive the mgfs of the following random variables:

(i) a discrete random variable X with the pmf $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, \ldots, n$ and $0 < p < 1$. (2 marks)

(ii) a discrete random variable X with the pmf $p(x) = \theta^x \exp(-\theta)/x!$ for $x = 0, 1, \ldots$ and $\theta > 0$. (2 marks)

(iii) a continuous random variable X with the pdf $f(x) = (1/\pi)(1 + x^2)^{-1}$ for $-\infty < x < \infty$. (2 marks)

(iv) a continuous random variable X with the pdf $f(x) = b^a x^{a-1} \exp(-bx)/\Gamma(a)$ for $x > 0$, $a > 0$ and $b > 0$, where $\Gamma(\cdot)$ denotes the gamma function. (2 marks)

(v) a continuous random variable X with the pdf $f(x) = 3x^2$ for $0 < x < 1$. (2 marks)

2. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $Uni[a, b]$, where both a and b are unknown.

(i) Write down the joint likelihood function of a and b. (2 marks)

(ii) Show that the maximum likelihood estimator (mle) of a is $\hat{a} = \min(X_1, X_2, \ldots, X_n)$. (1 marks)

(iii) Show that the mle of b is $\hat{b} = \max(X_1, X_2, \ldots, X_n)$. (1 marks)

(iv) Show that the mle, $\hat{a}$, is a biased and consistent estimator for a. (3 marks)

(v) Show that the mle, $\hat{b}$, is also a biased and consistent estimator for b. (3 marks)