MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 22 APRIL 2009

YOUR FULL NAME:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20% of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

1. For a random variable X let $M_X(t) = E[\exp(tX)]$ denote its moment generating function (mgf). Derive the mgfs of the following random variables:

(i) $X \sim Geom(\theta)$ with the pmf $p(x) = \theta(1-\theta)^{x-1}$ for $x = 1, 2, \ldots$. (2 marks)

(ii) $X \sim Po(\theta)$ with the pmf $p(x) = \theta^x \exp(-\theta)/x!$ for $x = 0, 1, \ldots$. (2 marks)

(iii) $X \sim Laplace(a, b)$ with the pdf $f(x) = (1/(2b)) \exp(- \mid x - a \mid /b)$ for $-\infty < x < \infty$. (2 marks)

(iv) $X \sim Exp(\lambda)$ with the pdf $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$. (2 marks)

(v) $X \sim Gumbel(\mu, \beta)$ with the pdf $f(x) = (1/\beta) \exp[-(x-\mu)/\beta] \exp\{-\exp[-(x-\mu)/\beta]\}$ for $-\infty < x < \infty$. (2 marks)

2. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown.

(i) Write down the joint likelihood function of μ and σ^2. (2 marks)

(ii) Show that the maximum likelihood estimator (mle) of μ is $\widehat{\mu} = \bar{X}$, where $\bar{X} = (1/n)\sum_{i=1}^{n} X_i$ is the sample mean. (2 marks)

(iii) Show that the mle of σ^2 is $\widehat{\sigma^2} = (1/n)\sum_{i=1}^{n}(X_i - \bar{X})^2$. (2 marks)

(iv) Show that the mle, $\widehat{\mu}$, is an unbiased and consistent estimator for μ. (2 marks)

(v) Show that the mle, $\widehat{\sigma^2}$, is a biased and consistent estimator for σ^2. (2 marks)