MATH20802: STATISTICAL METHODS
SECOND SEMESTER
IN CLASS TEST - 9 APRIL 2008

YOUR FULL NAME:

This test contains two questions. Please answer ALL of the questions. You must fully explain all your answers. This test will account for 20% of your final mark.

Each paper will be graded by myself. If you would have complaints about your mark please address them directly to me.

Good luck.

1. For a random variable X let $M_X(t) = E[\exp(tX)]$ denote its moment generating function (mgf). Derive the mgfs of the following random variables:

(i) $X \sim Bin(n, p)$ with the pmf $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, \ldots, n$. (2 marks)

(ii) X a discrete uniform random variable with the pmf $p(x) = 1/N$ for $x = 1, 2, \ldots, N$. (2 marks)

(iii) X a continuous uniform random variable with the pdf $f(x) = 1/(b-a)$ for $a < x < b$. (2 marks)

(iv) $X \sim Ga(a, \lambda)$ with the pdf $f(x) = \lambda^a x^{a-1} \exp(-\lambda x)/\Gamma(a)$ for $x > 0$. (2 marks)

(v) $X \sim N(\mu, \sigma^2)$ with the pdf $f(x) = (1/(\sqrt{2\pi}\sigma)) \exp\{-(x-\mu)^2/(2\sigma^2)\}$ for $-\infty < x < \infty$. (2 marks)

2. Consider the two independent random samples: $X_1, X_2, \ldots, X_n$ from $N(\mu_X, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_m$ from $N(\mu_Y, \sigma^2)$, where σ^2 is assumed known. The parameters μ_X and μ_Y are assumed not known.

(i) Write down the likelihood function of μ_X and μ_Y. (2 marks)

(ii) Find the maximum likelihood estimators (mles) of μ_X and μ_Y. (2 marks)

(iii) Find the mle of $\Pr(X < Y)$, where $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$ are independent random variables. (2 marks)

(iv) Show that the mle of μ_X in part (ii) is an unbiased and consistent estimator for μ_X. (2 marks)

(v) Show also that the mle of μ_Y in part (ii) is an unbiased and consistent estimator for μ_Y. (2 marks)