Revision

B2 (b)

$$F(x) = \frac{2}{\pi} \arcsin\left(\sqrt{x}\right), \quad -\infty < x < \infty$$

$$\omega(F) = +\infty$$

$$\text{I}: \quad \lim_{t \to \infty} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$= \lim_{t \to \infty} \frac{1 - \frac{2}{\pi} \arcsin\left(\sqrt{t + x\gamma(t)}\right)}{1 - \frac{2}{\pi} \arcsin\left(\sqrt{t}\right)}$$

$$\stackrel{LH}{=} \lim_{t \to \infty} \frac{\cancel{-\frac{2}{\pi}} \frac{1}{\sqrt{1 - t - x\gamma(t)}} \frac{(1 + x\gamma'(t))}{2\sqrt{t + x\gamma(t)}}}{\cancel{-\frac{2}{\pi}} \frac{1}{\sqrt{1-t}} \frac{1}{2\sqrt{t}}}$$

$$= \lim_{t \to \infty} \frac{\sqrt{1-t}}{\sqrt{1 - t - x\gamma(t)}} \left(1 + x\gamma'(t)\right) \frac{\sqrt{t}}{\sqrt{t + x\gamma(t)}}$$

$\neq e^{-x} \implies$ cond I does not hold.

II :
$$\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)}$$

$$= \lim_{t \to \infty} \frac{1 - \frac{2}{\pi} \arcsin\left(\sqrt{tx}\right)}{1 - \frac{2}{\pi} \arcsin\left(\sqrt{t}\right)}$$

$$\overset{LH}{=} \lim_{t \to \infty} \frac{\cancel{\left(-\frac{2}{\pi}\right)} \frac{1}{\sqrt{1 - tx}} \cdot \frac{\sqrt{x}}{2\sqrt{t}}}{\cancel{\left(-\frac{2}{\pi}\right)} \frac{1}{\sqrt{1-t}} \cdot \frac{1}{2\sqrt{t}}}$$

$$= \lim_{t \to \infty} \sqrt{\frac{1-t}{1-tx}} \cdot \sqrt{x}$$

$$= \lim_{t \to \infty} \sqrt{\frac{\frac{1}{t} - 1}{\frac{1}{t} - x}} \cdot \sqrt{x}$$

$$= \frac{1}{\sqrt{x}} \cdot \sqrt{x}$$

$$= 1 \neq x^{-\alpha}, \quad \alpha > 0$$

cond II does not hold.

III : does not hold because
$w(F) \not< \infty$

None of the 3 conditions hold.

Test 2016/2017

$$F(x) = 1 - \left\{1 - \left[1 - [1 - G(x)]^2\right]^3\right\}^4$$

Assume G belongs to Gumbel DoA.
Then

$$\lim_{t \to w(G)} \frac{1 - G(t + x\gamma(t))}{1 - G(t)} = e^{-x} \quad \ldots (*)$$

$$\Rightarrow \lim_{t \to w(F)} \frac{1 - F(t + x\gamma(t))}{1 - F(t)}$$

$$= \lim_{t \to w(G)} \frac{\left\{1 - \left[1 - \underbrace{[1 - G(t + x\gamma(t))]^2}_{\to 0}\right]^3\right\}^4}{\left\{1 - \left[1 - \underbrace{[1 - G(t)]^2}_{\to 0}\right]^3\right\}^4}$$

$$\boxed{(1-z)^3 \sim 1 - 3z \quad \text{as} \quad z \to 0}$$

$$= \lim_{t \to w(G)} \frac{\left\{\cancel{1} - \left[\cancel{1} - \cancel{3}[1 - G(t + x\gamma(t))]^2\right]\right\}^4}{\left\{\cancel{1} - \left[\cancel{1} - \cancel{3}[1 - G(t)]^2\right]\right\}^4}$$

$$= \lim_{t \to w(G)} \left[\frac{1 - G(t + x\gamma(t))}{1 - G(t)}\right]^8$$

$$\overset{(*)}{=} e^{-8x}$$

B1 (c)

$$F(x) = \frac{a\,G(x)\left[1+b-b\,G(x)\right]}{1-(1-a)\,G(x)\left[1+b-b\,G(x)\right]}$$

Assume G belongs to Gumbel DoA. Then there exist $\gamma(t) > 0$ such that

$$\lim_{t \to \omega(G)} \frac{1-G(t+x\gamma(t))}{1-G(t)} = e^{-x} \quad \ldots (*)$$

$$\Rightarrow \lim_{t \to \omega(F)} \frac{1-F(t+x\gamma(t))}{1-F(t)}$$

$$= \lim_{t \to \omega(G)} \frac{1-\dfrac{a\,G(t+x\gamma(t))\left[1+b-b\,G(t+x\gamma(t))\right]}{1-(1-a)\,G(t+x\gamma(t))\left[1+b-b\,G(t+x\gamma(t))\right]}}{1-\dfrac{a\,G(t)\left[1+b-b\,G(t)\right]}{1-(1-a)\,G(t)\left[1+b-b\,G(t)\right]}}$$

a a 1 1 1 1 1

1 1

$$= \lim_{t \to \omega(G)} \frac{1-G(t+x\gamma(t))\left[1+b-b\,G(t+x\gamma(t))\right]}{1-G(t)\left[1+b-b\,G(t)\right]}$$

1 1 1

$$= \lim_{t \to \omega(G)} \frac{1-G(t+x\gamma(t))}{1-G(t)} \overset{(*)}{=} e^{-x}.$$