Example

Suppose $\bar{F}(x, y) = (1 + ax + by + cxy)^{-q}$

for $x > 0, y > 0$.

Find G if it exists.

(i) $F_X(x) = 1 - \bar{F}(x, 0)$

$= 1 - (1 + ax)^{-q}$

$F_Y(y) = 1 - \bar{F}(0, y)$

$= 1 - (1 + by)^{-q}$

(ii) $w(F_X) = +\infty$.

$$\text{II} : \lim_{t \to \infty} \frac{1 - F_X(tx)}{1 - F_X(t)}$$

$$= \lim_{t \to \infty} \frac{1 - [1 - (1 + atx)^{-q}]}{1 - [1 - (1 + at)^{-q}]}$$

$$= \lim_{t \to \infty} \left(\frac{1 + atx}{1 + at} \right)^{-q}$$

$$= \lim_{t \to \infty} \left(\frac{\frac{1}{t} + ax}{\frac{1}{t} + a} \right)^{-q}$$

$$= x^{-q}$$

$\Rightarrow$ F_X belongs to Frechet domain

Similarly, F_Y belongs to Fréchet domain.

(iii) Set $F_X(x) = 1 - \frac{1}{n}$

$$\Rightarrow 1 - (1+ax)^{-q} = 1 - \frac{1}{n}$$

$$\Rightarrow (1+ax)^{-q} = \frac{1}{n}$$

$$\Rightarrow 1 + ax = n^{\frac{1}{q}}$$

$$\Rightarrow x = \frac{n^{\frac{1}{q}} - 1}{a}$$

$$\Rightarrow a_n = \frac{n^{\frac{1}{q}} - 1}{a}$$

$$\Rightarrow b_n = 0$$

$$\Rightarrow c_n = \frac{n^{\frac{1}{q}} - 1}{b}$$

$$\Rightarrow d_n = 0.$$

((iii)) $$\lim_{n\to\infty}\left[F(a_n x + b_n, c_n y + d_n)\right]^n$$

$$= \lim_{n\to\infty}\Big[1 - \bar{F}_X(a_n x + b_n, 0) - \bar{F}_Y(0, c_n y + d_n) + \bar{F}(a_n x + b_n, c_n y + d_n)\Big]^n$$

$$= \lim_{n\to\infty}\Big[1 - (1 + a(a_n x + b_n))^{-q} - (1 + b(c_n y + d_n))^{-q} + (1 + a(a_n x + b_n) + b(c_n y + d_n) + c(a_n x + b_n)(c_n y + d_n))^{-q}\Big]^n$$

$$= \lim_{n\to\infty}\Big[1 - (1 + (n^{\frac{1}{q}} - 1)x)^{-q} - (1 + (n^{\frac{1}{q}} - 1)y)^{-q} + (1 + (n^{\frac{1}{q}} - 1)x + (n^{\frac{1}{q}} - 1)y + \frac{c}{ab}(n^{\frac{1}{q}} - 1)^2 xy)^{-q}\Big]^n$$

$$\vdots$$

$$= e^{-x^{-q} - y^{-q}} = G(x,y).$$

$$= \lim_{n \to \infty} \left[1 - \frac{1}{n}\left(x + \frac{1-x}{n^{\frac{1}{q}}} \right)^{-q} \right.$$

$$- \frac{1}{n}\left(y + \frac{1-y}{n^{\frac{1}{q}}} \right)^{-q}$$

$$+ \frac{1}{n}\left(\frac{1}{n^{\frac{1}{q}}} + x + y - \frac{x}{n^{\frac{1}{q}}} - \frac{y}{n^{\frac{1}{q}}} \right.$$

$$\left. \left. + \frac{c}{ab} \frac{(n^{\frac{1}{q}} - 1)^2}{n^{\frac{1}{q}}} xy \right)^{-q} \right]^n$$

$\vdots$

$$= e^{-x^{-q} - y^{-q}}$$

$$= G(x, y).$$