Math48181/68181: Extreme Values and Financial Risk
Semester 1
Lecturer: Dr. Saralees Nadarajah

Office: Alan Turing 2.223.

Office Phone: 0161 275 5912.

Zoom link is `https://zoom.us/my/saraleesnadarajah`, you are most welcome to Zoom me 24 / 7.

Office Hours: Wednesday 11:00-12:00 and Fridays 11:00-12:00, you are most welcome to see me any other time, no appointments needed.

Skype id is "saraleesan", you are most welcome to skype me 24 / 7.

E-mail: mbbsssn2@manchester.ac.uk, you are most welcome to email me 24 / 7.

I am also happy to meet you in person anywhere 24 / 7 for tutoring.

WWW: `https://minerva.it.manchester.ac.uk/~saralees/extremes6.html`

Math48181/68181: Extreme Values and Financial Risk
Semester 1

Credit rating: 15.

Pre-requisite units: Statistical methods (Math20802).

Math48181/68181: Extreme Values and Financial Risk
Semester 1

Aims: To introduce probabilistic fundamentals and some statistical models in extreme value theory with applications to finance.

Brief description: The course will give some probabilistic and statistical details of univariate and bivariate extreme value theory. The topics covered will include: fundamental of univariate extreme value theory, the three extreme value distributions, various models for univariate extremes, fundamentals of bivariate extreme value theory, and various models for bivariate extremes. The course will contain a great deal material on applications of the models to finance. Software in R will be used.

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Intended Learning Outcomes

On successful completion of this unit students will: 1) have some understanding of the probabilistic fundamentals of univariate and bivariate extreme value theory; 2) be able to choose and fit appropriate extreme value models for a given data (univariate and bivariate); 3) be able to calculate probabilities associated with total portfolio loss, maximum portfolio loss and minimum portfolio loss; 4) be able to estimate financial risk measures; 5) be able to fit copulas to real data sets; 6) be able to fit GARCH type models to real data sets.

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Course Contents

I plan to cover all of the following topics:

1. Fluctuations of univariate maxima: the theory,
2. Fluctuations of univariate upper order statistics: the theory,
3. Some statistical models for univariate extremes,
4. Real data applications for univariate extremes using the R software,
5. Portfolio theory,
6. Real data applications,
7. Financial risk measures and their estimation,
8. Real data applications,
9. Models for stock returns,
10. Real data applications,
11. Some models for bivariate extremes,
12. Real data applications for bivariate extremes using the R software,
13. Copulas,
14. Real data applications,
15. GARCH type models,
16. Real data applications.

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Textbooks

Embrechts, P., Klüppelberg, C. and Mikosch, T. (1997) Modelling Extremal Events: for Insurance and Finance, Springer-Verlag, Berlin.

Leadbetter, M.R., Lindgren, G. and Rootzén, H. (1983) Extremes and Related Properties of Random Sequences and Processes, Springer-Verlag, Berlin.

Resnick, S.I. (1987) Extreme values, Regular Variation and Point Processes, Springer-Verlag, Berlin.

Coles S. (2001) An Introduction to Statistical Modelling of Extreme Values, Springer-Verlag, London.

Kotz, S. and Nadarajah, S. (2000) Extreme Value Distributions: Theory and Applications, Imperial College Press, London.

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Semester 1
Learning and Teaching Processes

One review session and one example class each week. In addition students are expected to do at least four hours private study each week on this course unit.

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Semester 1
Quizzes

There will be ten quizzes due at the following times:

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz1.pdf` **due by 11:00am on Wednesday, 13 October 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz2.pdf` **due by 11:00am on Wednesday, 20 October 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz3.pdf` **due by 11:00am on Wednesday, 27 October 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz4.pdf` **due by 11:00am on Wednesday, 10 November 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz5.pdf` **due by 11:00am on Wednesday, 17 November 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz6.pdf` **due by 11:00am on Wednesday, 24 November 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz7.pdf` **due by 11:00am on Wednesday, 1 December 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz8.pdf` **due by 11:00am on Wednesday, 8 December 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz9.pdf` **due by 11:00am on Wednesday, 15 December 2021**

`https://minerva.it.manchester.ac.uk/~saralees/ext2021quizz10.pdf` **due by 11:00am on Wednesday, 5 January 2022**
Each quiz will be worth 1 percent.

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In-class test

There will be an in-class test on Friday 17 December 2021 accounting for 10 percent. The formulas you will need to remember for this test are in

https://minerva.it.manchester.ac.uk/~saralees/cwformula2021.pdf

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Final exam

The final exam for this course will be in January 2022, the formulas you will need to remember for this exam are in

https://minerva.it.manchester.ac.uk/~saralees/formula202120.pdf

The final exam will account for 80 percent of your final mark.

"Extreme Values" & "Financial Risk"

- Extreme values can cause financial risk
- Financial risk can be caused by extreme values

- Some parts of this course will focus on extreme values
- Other parts of this course will focus on financial risk

What is the use of this course?

1) try to minimise the chance of financial disasters

2) to determine fradulent activities when they occur

FINANCIAL DISASTERS

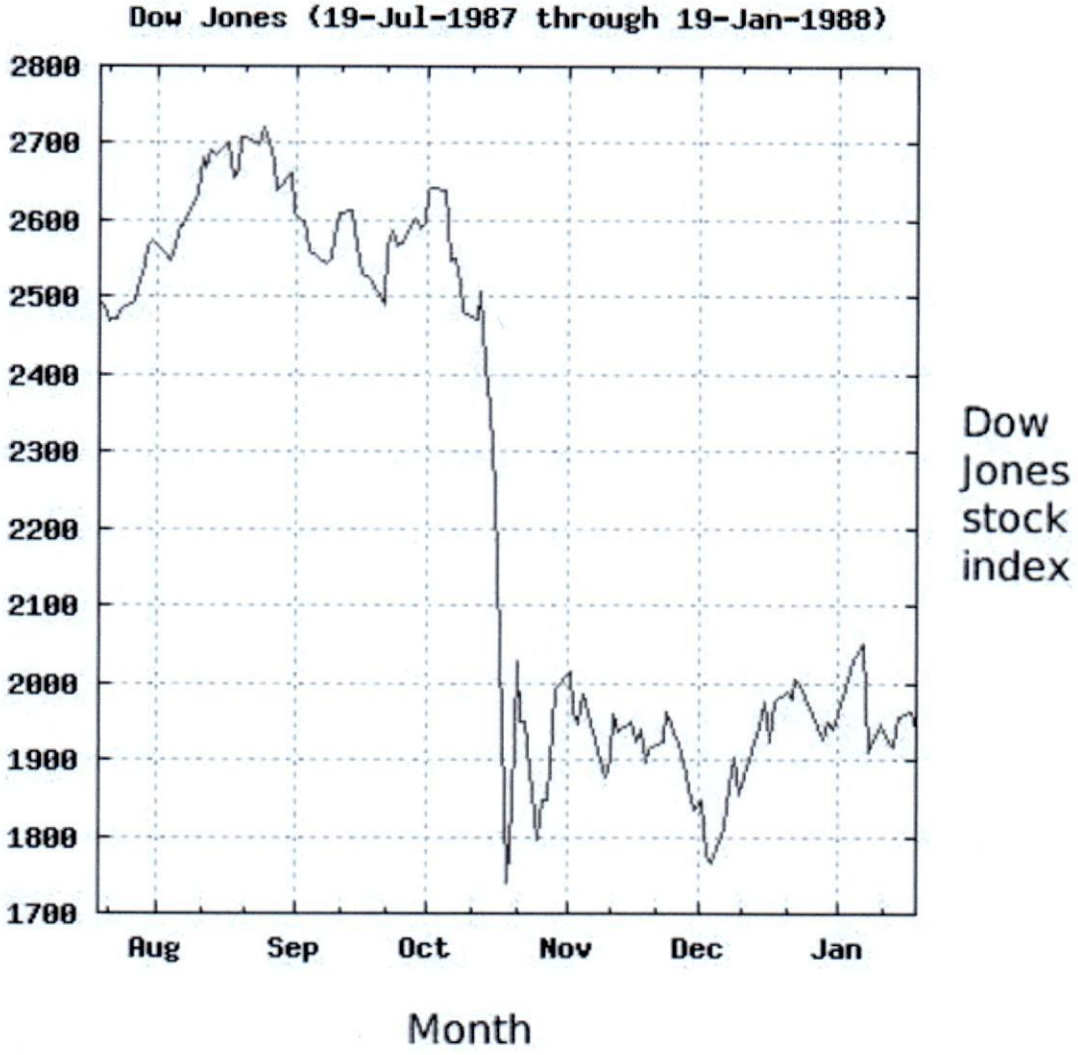


Figure 1: Black Monday crash on 19 October 1987. The Dow Jones stock index crashed down by 22.6 percent (by 508 points). Overall the stock market lost $0.5 trillion.

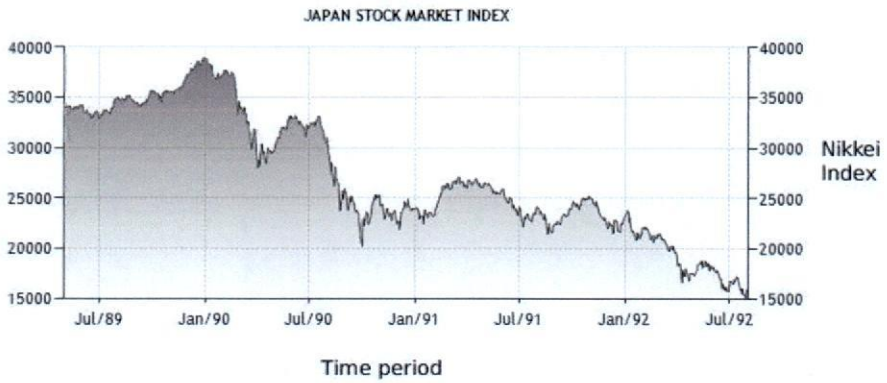


Figure 2: Japan stock price bubble near the end of 1989. A loss of $2.7 trillion in capital. A recovery happened after mid-1990.

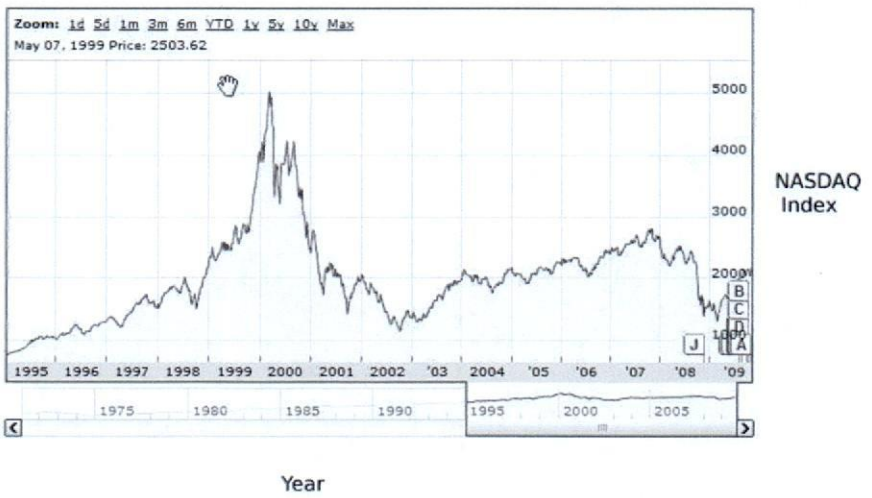


Figure 3: Dot com bubble (the NASDAQ index) during 1999 and 2000. The bubble burst on 10 March 2000. The peak on that day was \$5048.62. There is a recovery after 2002. Never recovered to attain the peak.

Figure 4: Asian financial crisis (Asian dollar index) in July 1997. Not fully recovered even in 2011.

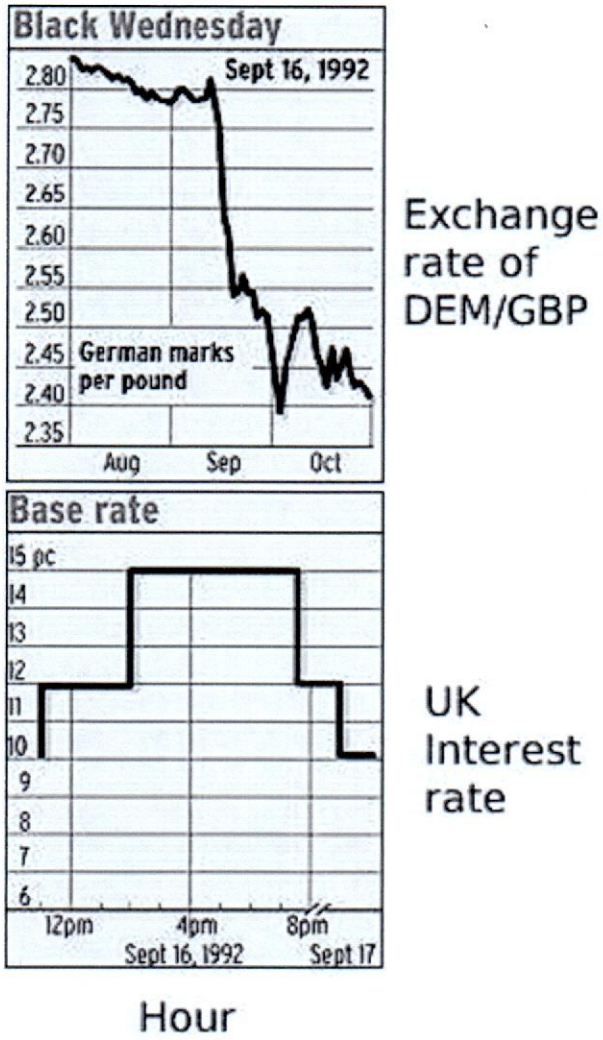
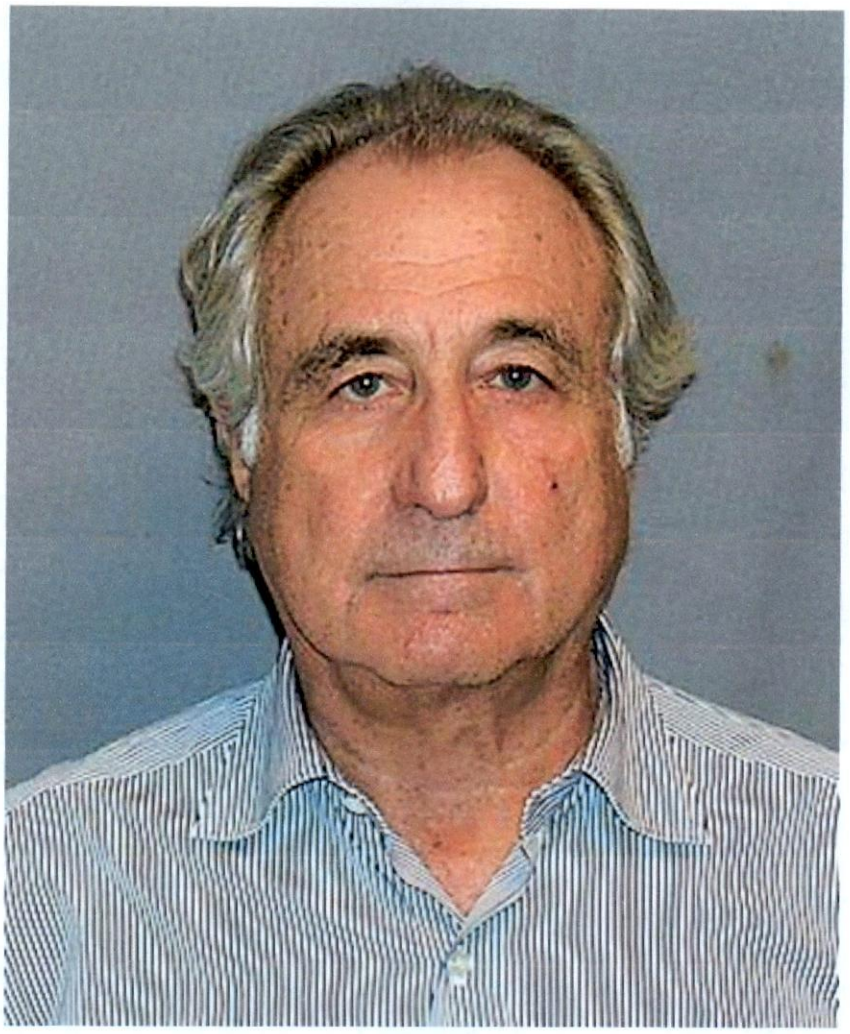


Figure 5: Black Wednesday crash of 16 September 1992. Top image shows the exchange rate of Deutsche mark to British pounds. Bottom image shows the UK interest rate on the day.

The biggest financial frauds

There have been many financial frauds in the world. The frauds have been the result of insider trading info or massive Ponzi schemes. Some of the most well known fraudsters are shown in the following pages.



Bernard Madoff

Former stockbroker and investment adviser. His name is connected to the most well known Ponzi scheme fraud. He earned $62 billion through such fraud and is currently sentenced to a maximum of 150 years in prison.

Albert H. Wiggin

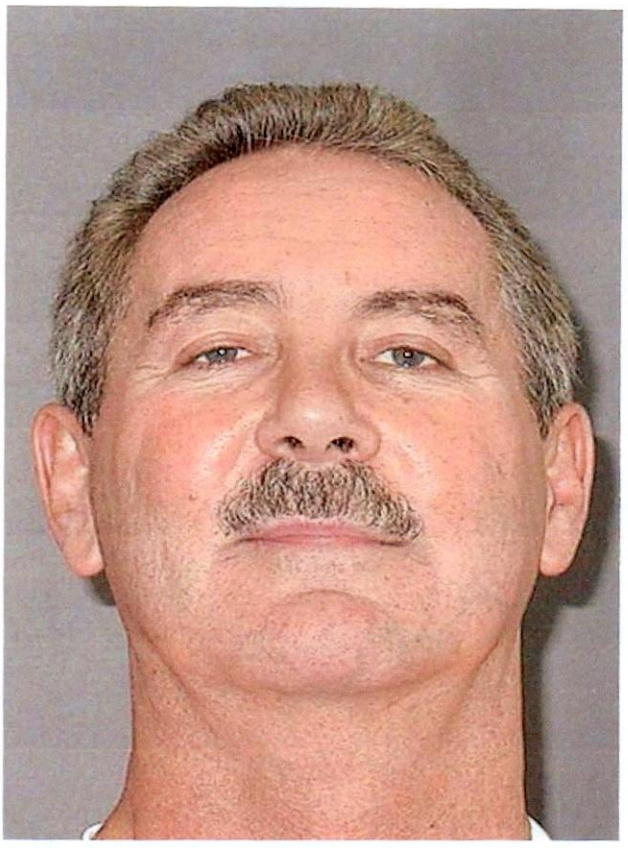
Career spaning from head of Chase National Bank and now JP Morgan chase. He shorted his own company with 40000 shares in 1929 and profited immensely to the tune of $4 million and wasn't even found guilty of breaking any laws.

Bruno Iksil

Former trader for JPMorgan Chase & Co and also known as "the London Whale". His position in the credit default swap market caused more than $6.2 billion in losses for the firm in 2012.

Jordan Belfort

Belfort founded the brokeage firm Stratton Oakmont, and went on to defraud investors through sales of stock while employing more than 1,000 people. The firm was shut in 1998. Belfort was charged with money laundering and fraud.



Allen Stanford

Stanford is serving a 110 year prison sentence, having been convicted of charges of fraud. Stanford was the chairman of the now defunct Standord Financial Group of companies.

Jeff Skilling

Jeff was the CEO of Enron Corp. He was convicted of federal felony charges relating to Enron's financial collapse and is currently serving 14 years prison sentence.

Ivan Boesky

Boesky is a former American stock trader who is notable for his prominent role in a Wall Street insider trading scandal that occurred in the US in the mid-1980s. In 1987, Boesky was sentenced to 3 years in prison.

Barry Minkow

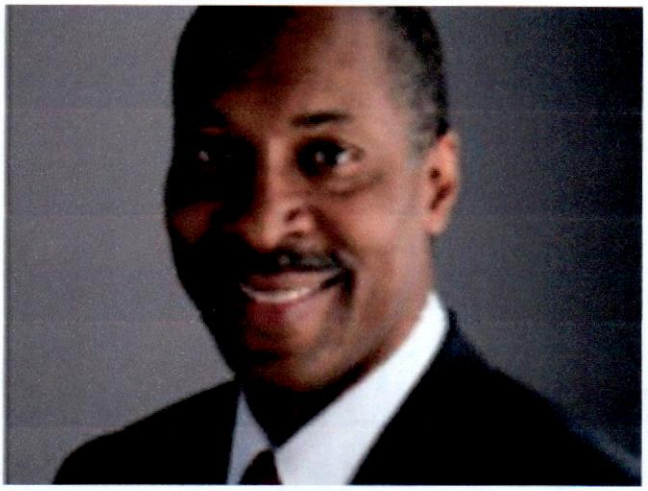
Founded ZZZZ Best which appeared to be an immensely successful carpet cleaning and restoration company. However, it was actually a front to attract investment for a massive Ponzi scheme. It collapsed in 1987 costing investors and lenders $100 million.

Martha Stewart

Martha is an American businesswoman, writer and television personality. In 2004, Stewart was convicted relating to the ImClone insider trading affair and sentenced to prison.

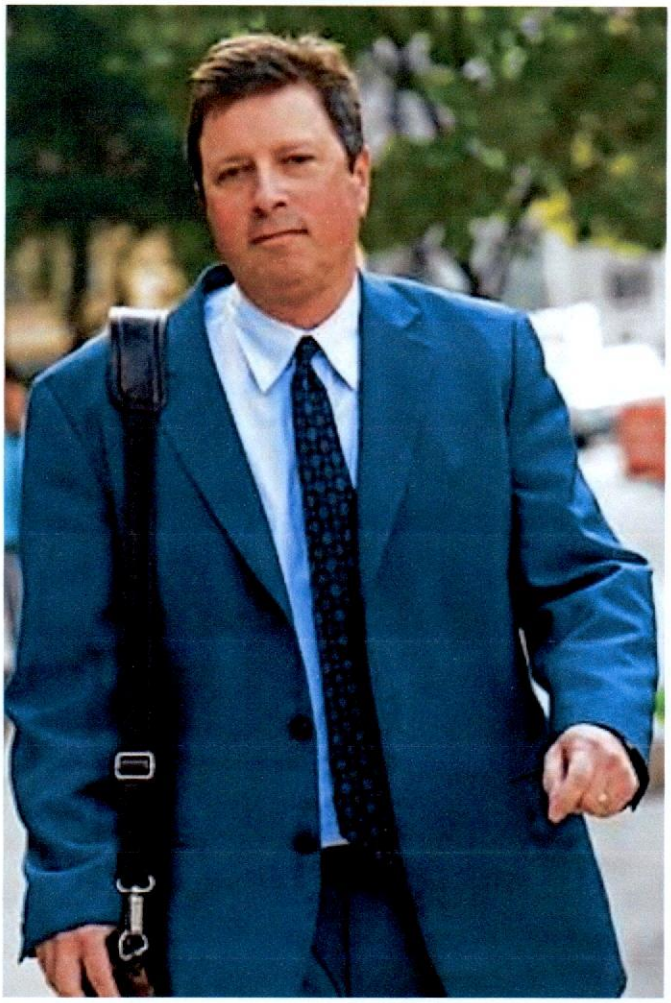
Bernard Ebbers

A Canadian born buisnessman and cofounded the telecommunications company WorlCom. In 2005, he was convicted of fraud and conspiracy as a result of WorldCom's false financial reporting. He was sentenced to 25 years for his role in an $11 billion accounting fraud. It is the largest accounting scandal in US history.



Donald Johnson

Executive of stock exchange for Nasdaq's. Convicted in 2011 of using insider information to trade shares of United Therapeutics, Honda and other companies from 2006 to 2009. He was sentenced to 3 and half years in prison for a total of $755,000 of fraud.



Doug Whitman

Managing more than $100 million through his Whitman Capital hedge fund. He was convicted in August 2012 of insider trading charges involving stocks of Google, Polycom and Marvell Technology Group. The total value of his fraud accumulated to $1 million and lead to 2 years in prison.

Raj Rajaratnam

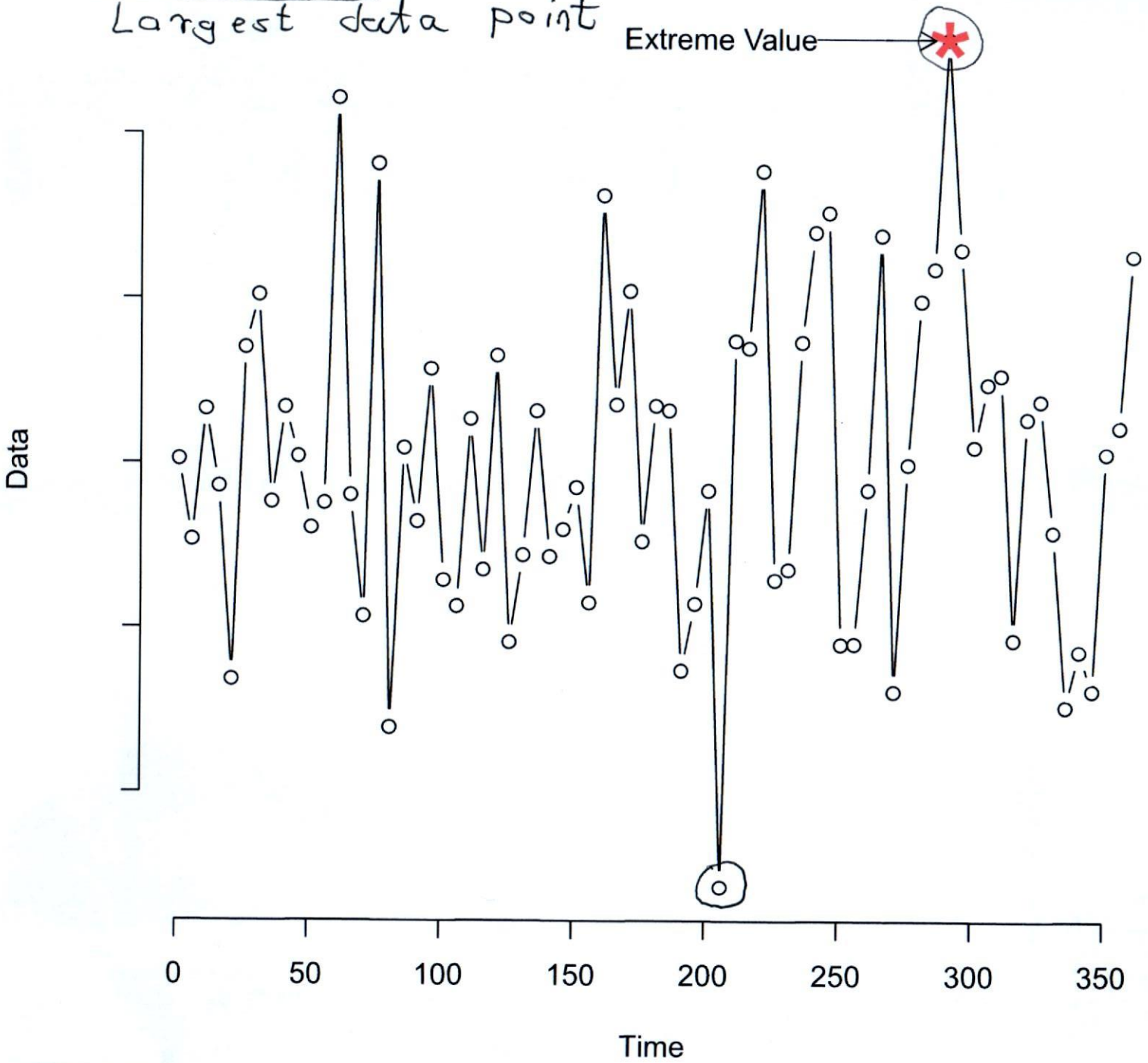
Former head of Galleon Capital. He received the longest insider trading sentence in history of 11 years for securities fraud and five counts of conspiracy. He was ordered to pay back the $53.8 million of fraud.

What is an extreme value?

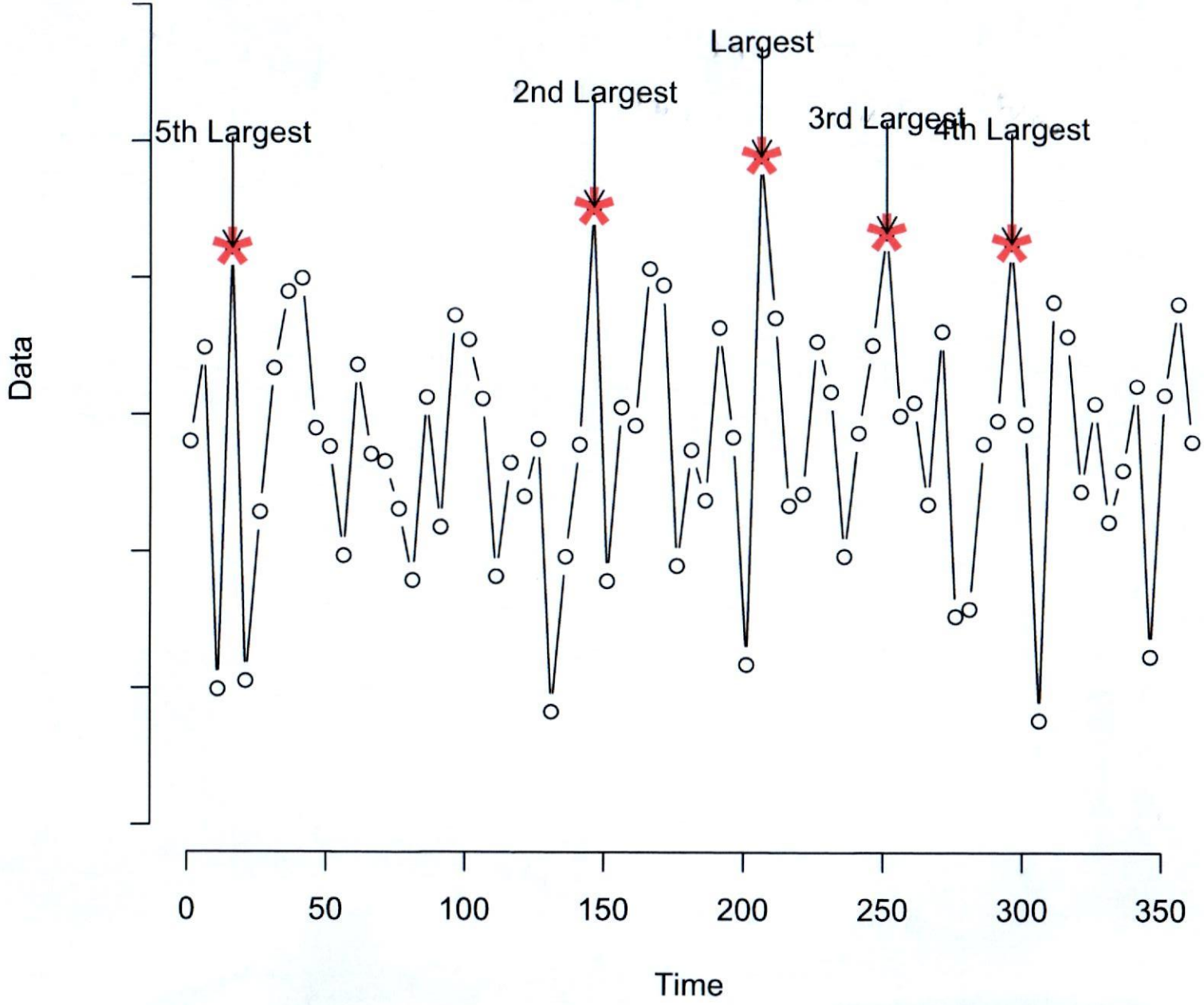
There are many ways to define an extreme value.

In this course, I will discuss 3 definitions of an extreme value.

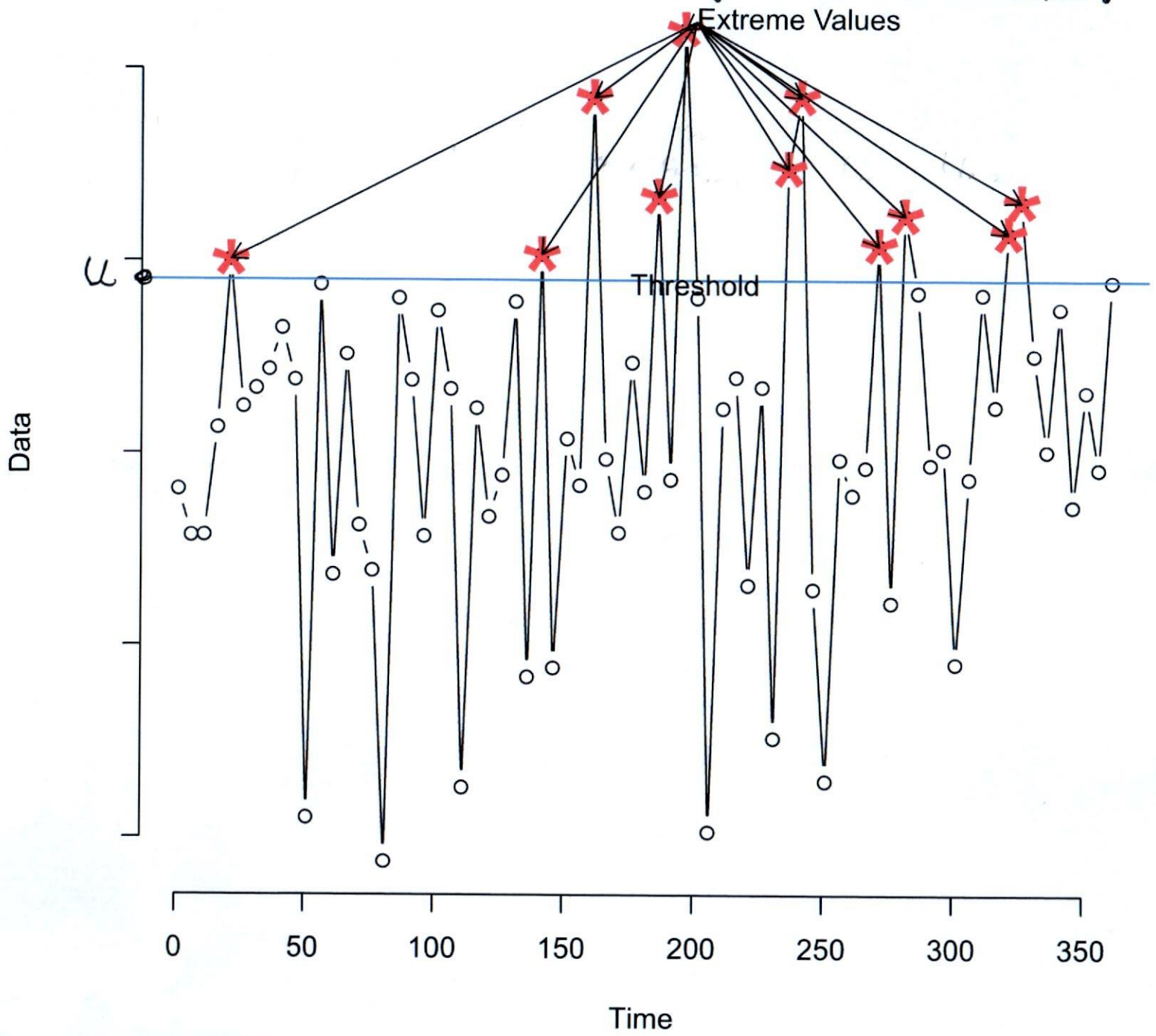
Definition 1
Largest data point



Definition 3 : Extreme values
= r largest data values



Definition 2 Extreme values are data exceeding the threshold.



Distribution of extreme value under definition 1

Suppose $\underbrace{X_1, \ldots, X_n}_{\text{data}}$ are independently and identically distributed with CDF $F(\cdot)$

(CDF: Cumulative distribution function)

By definition 1, the extreme value is

$$M_n = \max(X_1, \ldots, X_n).$$

The CDF of M_n is

$$\begin{aligned} & P(M_n \leq x) \\ &= P(\max(X_1, \ldots, X_n) \leq x) \\ &= P(X_1 \leq x, \ldots, X_n \leq x) \\ &\overset{\text{indep}}{=} P(X_1 \leq x) \cdots P(X_n \leq x) \\ &= F(x) \cdots F(x) \\ &= [F(x)]^n \end{aligned}$$

People are usually interested in the behavior of M_n over large periods. That is, what is the distribution of M_n as $n \to \infty$?

$$\lim_{n \to \infty} P(M_n \leq x)$$

$$= \lim_{n \to \infty} [F(x)]^n$$

$$= \begin{cases} 0 & \text{if } 0 \leq F(x) < 1 \\ 1 & \text{if } F(x) = 1 \end{cases}$$

This is not a useful result for practical applications.

Suppose $X_1, \ldots, X_n$ are independent and identical with $E(X_i) = \mu$ and $\text{Var}(X_i) = \sigma^2$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample mean.

1) $\lim_{n \to \infty} \bar{X} = \mu$ "Strong Law of Large Numbers"

Not a very useful result

scaling

2) $\lim_{n \to \infty} \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = N(0, 1)$ "Central limit theorem" (CLT)

CLT is a very useful result

We look at the limit of

$$P\left(\frac{M_n - b_n}{a_n} \leq x\right).$$

$$\Rightarrow \lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right)$$

$$= \lim_{n\to\infty} P(M_n \leq b_n + a_n x)$$

$$= \lim_{n\to\infty} P(\max(X_1, \dots, X_n) \leq b_n + a_n x)$$

$$= \lim_{n\to\infty} P(X_1 \leq b_n + a_n x, \dots, X_n \leq b_n + a_n x)$$

$$\overset{\text{indep}}{=} \lim_{n\to\infty} P(X_1 \leq b_n + a_n x) \cdots P(X_n \leq b_n + a_n x)$$

$$= \lim_{n\to\infty} F(b_n + a_n x) \cdots F(b_n + a_n x)$$

$$= \lim_{n\to\infty} [F(b_n + a_n x)]^n \quad \cdots \quad (*)$$

What is the limit of $(*)$?

Hint for quiz 1

$$\operatorname{sech}(x) = \frac{2}{e^{x} + e^{-x}}$$

L'Hopital's Rule

$$\lim_{x \to \infty} \frac{g(x)}{h(x)} = \lim_{x \to \infty} \frac{g'(x)}{h'(x)}$$

provided that

$$\lim_{x \to \infty} g(x) = \pm \infty \quad (0)$$

and

$$\lim_{x \to \infty} h(x) = \pm \infty \quad (0)$$