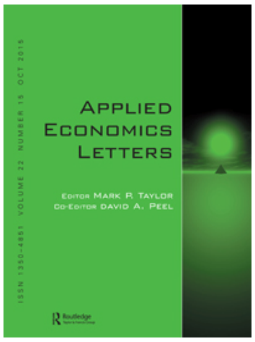

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Extreme value analysis of electricity demand in the UK

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For the first time, an extreme value analysis of electricity demand in the UK is provided. The analysis is based on the generalized Pareto distribution. Its parameters are allowed to vary linearly and sinusoidally with respect to time to capture patterns in the electricity demand data. The models are shown to give reasonable fits. Some useful predictions are given for the value at risk of the returns of electricity demand.

Keywords: extreme values; generalized Pareto distribution; value at risk

JEL Classification: C16; C13

I. Introduction

Electricity demand for the UK has been studied by many authors: Psiloglou *et al.* (2009) identified temperatures, thermal comfort levels, weekly effects, holiday effects and other economic, social and demographic factors as explaining electricity demand in London, UK; Papadopoulos *et al.* (2011) studied electricity demand with electric cars in 2030 for the UK; Al-Qahtani and Crone (2013) proposed a multivariate k nearest neighbour regression method for forecasting electricity demand for the UK.

Often what is of interest is the peak of electricity demand. As pointed out by Sigauke *et al.* (2013), peak electricity demand modelling

> is a policy concern for countries throughout the world. Many countries are investing heavily in the construction of new (reserve) generating plants in order to increase electricity supply during peak demand periods.

Statistical analysis of peaks involves the use of extreme value models. We are not aware of any study applying extreme value models for electricity demand in the UK. Even globally, there have been only a few studies applying extreme value models for electricity demand. The ones we are aware of are the study of Chikobvu and Sigauke (2013) that modelled the influence of temperature on average daily electricity demand in South Africa using a piecewise linear regression model and the generalized extreme value theory; and the study of Sigauke *et al.* (2013) that modelled extreme daily increases in peak electricity demand for South Africa using the generalized Pareto distribution.

However, there have been studies applying extreme value models to other aspects of electricity: electricity spot price modelling at work for the EEX Phelix Base electricity price index (Klüppelberg *et al.*, 2010); modelling of electricity pool prices from the Australian National Electricity Market (Dev and Martin, 2014); modelling of extreme events in electricity spot markets in Australia (Herrera and Gonzalez, 2014).

This is the first article modelling extreme values of electricity demand for the UK. We use the

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generalized Pareto distribution to model the extreme values as in Sigauke *et al.* (2013) and Dev and Martin (2014). But one important distinction is that our models explain how the extremes of electricity demand vary with respect to time. This feature was not accommodated in the models considered in Sigauke *et al.* (2013) and Dev and Martin (2014).

The use of the generalized Pareto distribution to model extreme values is theoretically motivated as stated in Section II. There are other distributions for modelling extreme values like the generalized extreme value distribution used in Chikobvu and Sigauke (2013). But the use of this distribution wastes data as well known. The generalized Pareto distribution uses much more of the data.

The contents of this article are organized as follows: Models based on the generalized Pareto distribution and procedures for fitting to the UK electricity demand data (Section II, Chan and Nadarajah, 2014) are discussed in Section II. The results based on the fitting of these models are discussed in Section III. Finally, some conclusions are noted in Section IV.

II. Models

Let X denote a random variable. We say X takes an extreme value if $X > u$ for some high threshold u. Extreme values in this context could be high values of the returns of electricity demand or high values of the negative returns of electricity demand.

Pickands (1975) developed much asymptotic theory for the extreme values X. According to that theory, if certain regularity conditions are satisfied and u is sufficiently large then

$$\Pr(X > x + u | X > u) \approx \left[1 + \xi \frac{x}{\sigma}\right]^{-1/\xi}, \qquad (1)$$

where $\sigma > 0$ is the scale parameter, $-\infty < \xi < \infty$ is the shape parameter and $1 + \xi x/\sigma > 0$. Rearranging Equation 1, we can express the distribution function of X as

$$F(x) = \Pr(X < x) \approx 1 - p\left[1 + \xi \frac{x-u}{\sigma}\right]^{-1/\xi} \qquad (2)$$

for $u \le x < \infty$ if $\xi \ge 0$ and $u \le x \le u + \sigma/\xi$ if $\xi < 0$, where $p = \Pr(X > u)$. The model given by Equation 2 is known as the generalized Pareto model.

Chan and Nadarajah (Section II, 2014) noted that the extreme returns of electricity demand show some evidence of seasonality and trend. To see if seasonality is significant we fitted the following models:

- Model S1: X is distributed according to Equation 2 with σ fixed and ξ fixed;
- Model S2: X is distributed according to Equation 2 with $\sigma = \exp[a + b\sin(\pi M/12) + c\cos(\pi M/12)]$ and ξ fixed, where M denotes the month number (1 for January and 12 for December);
- Model S3: X is distributed according to Equation 2 with σ fixed and $\xi = d + e\sin(\pi M/12) + f\cos(\pi M/12)$, where M denotes the month number (1 for January and 12 for December);
- Model S4: X is distributed according to Equation 2 with $\sigma = \exp[a + b\sin(\pi M/12) + c\cos(\pi M/12)]$ and $\xi = d + e\sin(\pi M/12) + f\cos(\pi M/12)$, where M denotes the month number (1 for January and 12 for December).

To see if trend is significant we fitted the following models:

- Model T1: X is distributed according to Equation 2 with σ fixed and ξ fixed;
- Model T2: X is distributed according to Equation 2 with $\sigma = \exp[g + h(Y - 2010)]$ and ξ fixed, where Y denotes the year;
- Model T3: X is distributed according to Equation 2 with σ fixed and $\xi = s + t(Y - 2010)$, where Y denotes the year;
- Model T4: X is distributed according to Equation 2 with $\sigma = \exp[g + h(Y - 2010)]$ and $\xi = s + t(Y - 2010)$, where Y denotes the year.

Note that S1 and T1 are the same models.

Each model was fitted by the method of maximum likelihood. Discrimination among the fitted models S1–S4 (and that among the fitted models T1–T4) was performed using the likelihood ratio test (Cox and Hinkley, 1974). The value of u was chosen by plotting the empirical estimate of $E(X - u|X > u)$ versus u known as the mean residual life plot.

III. Results and Discussion

The mean residual plots for the returns and the negative returns are shown in Fig. 1. The u for the returns (respectively, negative returns) can be chosen as $u = 0.115$ (respectively, $u = 0.076$). Hence, high returns (respectively, negative low returns) can be taken as those returns exceeding 0.115 (respectively, 0.076).

The auto correlation function plot of all returns (respectively, all negative returns) exceeding 0.115 (respectively, 0.076) is shown in Fig. 2. This figure shows evidence to support the fact that all the high returns (or, all the negative low returns) are independent. We also tested for no serial correlation using Durbin and Watson's method. This gave the p-values of 0.12 and 0.43 for high returns and negative low returns, respectively. Hence, the models in Section II can be fitted to all high returns as well as all negative low returns.

Models S1–S4 and T1–T4 were fitted by the method of maximum likelihood. The parameter estimates, SE and the log-likelihoods for the high and low returns are shown in Tables 1 and 2 of Chan and Nadarajah (2014). The SEs were computed by inverting the observed information matrix of the maximum likelihood estimates (Cox and Hinkley, 1974).

The best model among models from T1 to T4 (or, S1 to S4) as shown in Table 1 of Chan and Nadarajah (2014), is model T2 (or, S2), which is determined by the standard likelihood ratio test. This shows that there is a significant downward trend and a significant seasonality in the scale of the high returns. We also fitted a model that combines the trend and seasonality:

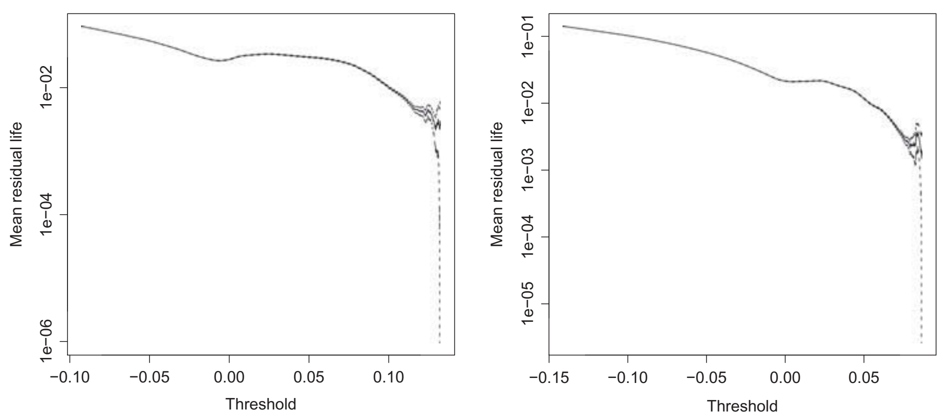

Fig. 1. Mean residual plots for the returns (left) and the negative returns (right). The *y*-axes are in log scale

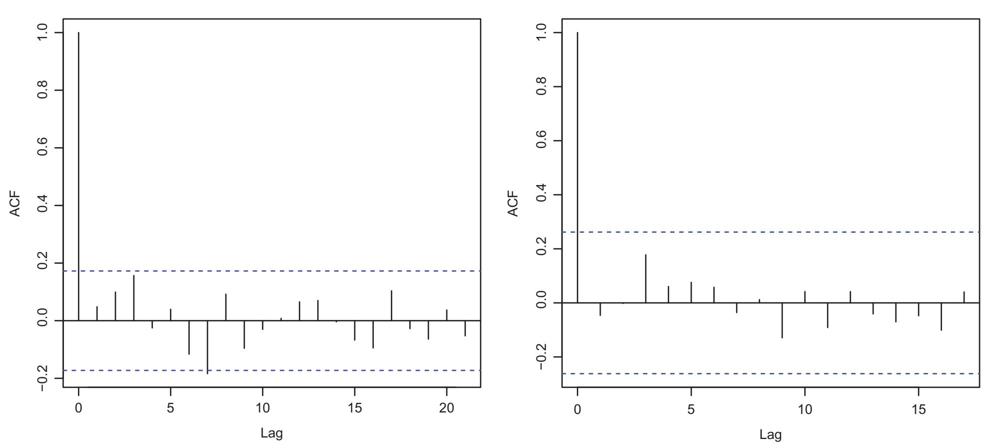

Fig. 2. Auto correlation function plots of the high returns (left) and the negative low returns (right)

> X is distributed according to Equation 2 with $\sigma = \exp[a + b\sin(\pi M/12) + c\cos(\pi M/12)] + d(Y - 2010)$ and ξ fixed, where M denotes the month number (1 for January and 12 for December) and Y denotes the year.

We refer to this model as S2T2. The parameter estimates were $\hat{\xi} = -1.754 \times 10^{-1}(4.717 \times 10^{-2})$, $\hat{a} = -5.854(2.271 \times 10^{-1})$, $\hat{b} = 8.322 \times 10^{-1}(2.208 \times 10^{-1})$, $\hat{c} = -4.779 \times 10^{-1}(1.032 \times 10^{-1})$, $\hat{d} = -5.210 \times 10^{-4}(2.790 \times 10^{-4})$ with log $L = 1508.758$. Clearly, model S2T2 is a significant improvement on models S2 and T2. Hence, it can be chosen as the best model for high returns. Using Table 2 of Chan and Nadarajah (2014), the best model for negative low returns was chosen, that is, model S3. The probability and quantile plots for the best fitting models (S2T2 for high returns and S3 for negative low returns) are shown in Figs 3 and 4. The plots show that the best fitting models describe the data reasonably.

The estimates of value at risk with probability q for the two best fitting models are

$$\mathrm{VaR}_q \approx u + \frac{1}{\hat{\xi}}\left\{\exp\left[\hat{a} + \hat{b}\sin\left(\frac{\pi M}{12}\right) + \hat{c}\cos\left(\frac{\pi M}{12}\right)\right] + \hat{d}(Y - 2010)\right\}\left[p^{\hat{\xi}}(1-q)^{-\hat{\xi}} - 1\right]$$

and

$$\mathrm{VaR}_q \approx u + \frac{\hat{\sigma}}{\hat{d} + \hat{e}\sin(\pi M/12) + \hat{f}\cos(\pi M/12)}\left[\left(\frac{p}{1-q}\right)^{\hat{d} + \hat{e}\sin(\pi M/12) + \hat{f}\cos(\pi M/12)} - 1\right].$$

The former is the value at risk for high returns. The latter is the value at risk for negative low returns. The

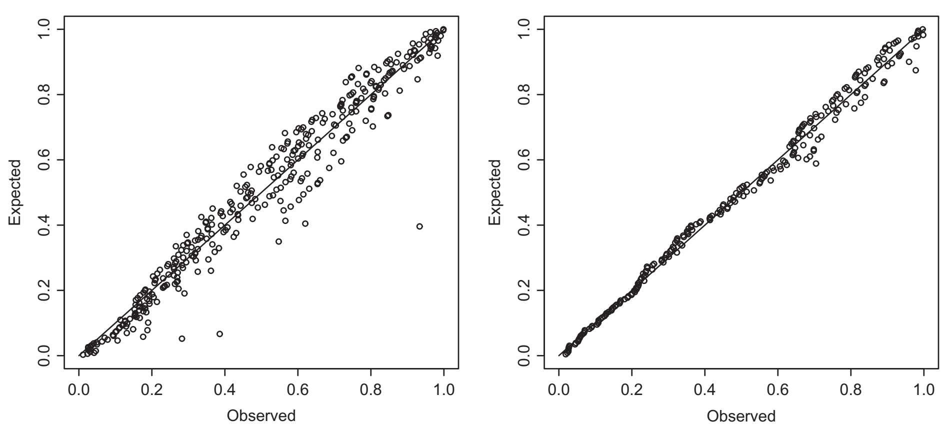

Fig. 3. Probability plots for the high returns (left) and the negative low returns (right)

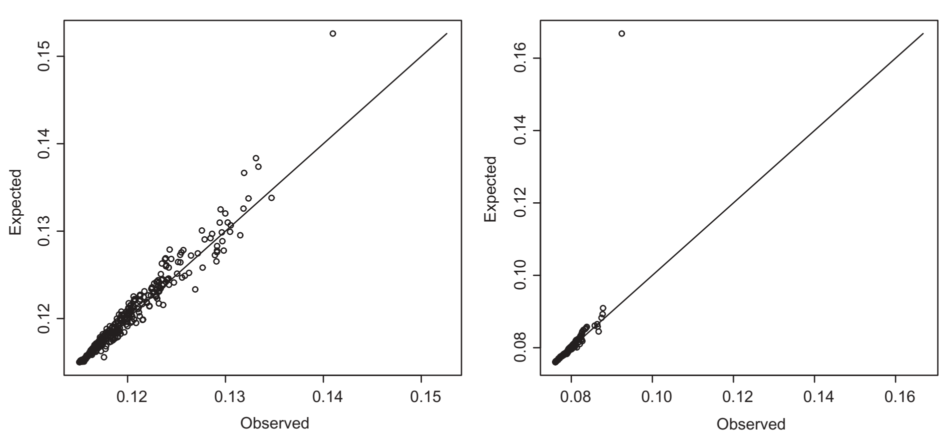

Fig. 4. Quantile plots for the high returns (left) and the negative low returns (right)

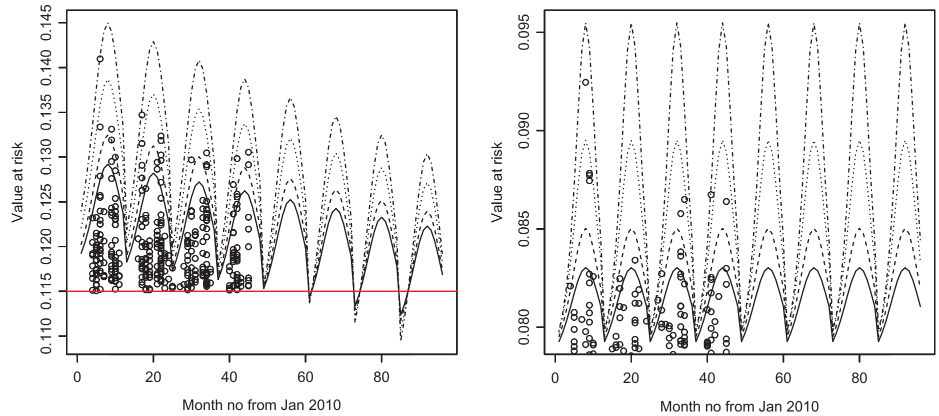

Fig. 5. Values at risk for the high returns (left) and the negative low returns (right). The solid curve is for $q = 0.9$, the dashed curve is for $q = 0.95$, the dotted curve is for $q = 0.99$ and the dashed and dotted curve is for $q = 0.999$

plots of these estimates from January 2010 to December 2017 are shown in Fig. 5. Also shown in the plots are the observed data from January 2010 to September 2013. The predictions given from October 2013 to December 2017 can be useful for practitioners. In particular, they can be useful for policymakers about how extreme day to day changes in electricity demand can become. Such predictions based on value-at-risk estimates for electricity spot markets in Australia are given in Herrera and Gonzalez (2014).

The value at risk for high returns appears largest for summer months and smallest for winter months. The value at risk for negative low returns also appears largest for summer months and smallest for winter months. To the best of our knowledge, these findings are new and have not been noted by other researchers. The common finding of other researchers has been that the electricity demand (not its returns) is largest for winter months and smallest for summer months. Our findings are in sharp contrast (but do not contradict the common finding) and can be useful for policymakers with respect to electricity demand.

IV. Conclusions

We have used models based on the generalized Pareto distribution to explain how the extremes of the returns of electricity demand in the UK vary with respect to time. Some of the main conclusions are as follows: the high returns of electricity demand show a downward trend in scale and seasonality in scale; the negative low returns of electricity demand show seasonality in shape but no significant trends.

A future work is to extend the models in Section II to accommodate factors like temperature and holiday effects. Another is to consider neural networks and time series models for electricity demand in the UK.

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