LECTURE

MONDAY 11 FEB

15:00-16:00

MATH10802

Z-table

This table gives the cumulative distribution function of a standard normal distribution, i.e. $\Phi(z) = P(Z \leq z)$, where $Z \sim N(0,1)$.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Example 2

We want to compute

$$P(X > 3)$$

$$= P\left(\frac{X-1}{2} > \frac{3-1}{2}\right)$$

$$= P(N(0,1) > 1)$$

$$= 1 - P(N(0,1) \leq 1)$$

$$= 1 - \Phi(1)$$

$$= 1 - 0.8413$$

Example 3

Suppose $X \sim N(1, 4)$

$$P(X < -1)$$

$$= P\left(\frac{X-1}{2} < \frac{-1-1}{2}\right)$$

$$= P(N(0,1) < -1)$$

$$= \Phi(-1)$$

$$= 1 - \Phi(1)$$

$$= 1 - 0.8413$$

$$\Phi(-x) = 1 - \Phi(x)$$

Example 4

Suppose $X \sim N(1, 4)$

$$P(-1 < X < 4)$$

$$= P\left(\frac{-1-1}{2} < \frac{X-1}{2} < \frac{4-1}{2}\right)$$

$$= P(-1 < N(0,1) < 1.5)$$

$$= P(N(0,1) < 1.5) - P(N(0,1) < -1)$$

$$= \Phi(1.5) - \Phi(-1)$$

$$= \Phi(1.5) - [1 - \Phi(1)]$$

$$= \Phi(1.5) - 1 + \Phi(1)$$

$$= 0.9332 - 1 + 0.8413$$

Binomial Distribution

A RV X has a binomial distribution

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k},$$

$$k = 0, 1, \ldots, n$$

$$0 \leq p \leq 1$$

n = size parameter

p = prob of success.

$X \sim \text{Binom}(n, p)$

The CDF of

$X \sim Binom(n, p)$ is

$$F_X(k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$$

In R,

dbinom(x, size = n, prob = p)

will compute PDF

pbinom(x, size = n, prob = p)

will compute CDF

Properties If $X \sim \text{Binom}(n, p)$

1. $E(X) = np$

2. $\text{Var}(X) = np(1-p)$.

Example 1

Suppose $X \sim \text{Binom}\left(5, \frac{1}{2}\right)$

$$P(X < 1)$$

$$= P(X = 0)$$

$$= \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(1 - \frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5$$

Example 2

Suppose $X \sim \text{Binom}\left(5, \frac{1}{2}\right)$

$$P(1.5 < X < 3.5)$$

$$= P(X = 2) + P(X = 3)$$

$$= \binom{5}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$$

$$+ \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$

Example 3 Suppose

$X \sim Binom(100, 0.1)$

$$P(X = 2)$$

$$= \binom{100}{2} (0.1)^2 (1-0.1)^{98}$$

$$P(91 \leq X \leq 94)$$

$$= \sum_{i=91}^{94} \binom{100}{i} (0.1)^i (0.9)^{100-i}$$

Is there a simple way?

Central Limit Thm (CLT)

Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a population with $E(X) = \mu$ & $Var(X) = \sigma^2$.

Then

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \longrightarrow N(0, 1) \quad -(*)$$

as $n \to \infty$, where

$$\bar{X} = \frac{X_1 + \cdots + X_n}{n}.$$

If n is a large number

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \approx N(0,1)$$

$$\Rightarrow \bar{X} - \mu \approx \frac{\sigma}{\sqrt{n}} \cdot N(0,1)$$

$$\equiv N\left(0, \frac{\sigma^2}{n}\right)$$

$$\Rightarrow \bar{X} \approx \mu + N\left(0, \frac{\sigma^2}{n}\right)$$

$$\Rightarrow \bar{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\Rightarrow \frac{\sum_{i=1}^{n} X_i}{n} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\Rightarrow \sum_{i=1}^{n} X_i \approx n \cdot N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\equiv N(n\mu, n\sigma^2)$$

$$\Rightarrow \sum_{i=1}^{n} X_i \approx N(n\mu, n\sigma^2).$$

... (**)

Suppose $X_1, X_2, \ldots, X_n$ are IID Bernoulli (p) RVs.

Then

$$\sum_{i=1}^{n} X_i \sim \text{Binomial}(n, p)$$

For the Bernoulli,

$$\mu = E(X) = p$$

and $\sigma^2 = \text{Var}(X) = p(1-p)$.

By (**),

$$\sum_{i=1}^{n} X_i \approx N(np, np(1-p))$$

$$\Rightarrow \text{Binomial}(n, p)$$

$$\approx N(np, np(1-p)).$$