# MATH10282: INTRODUCTION TO STATISTICS
# SEMESTER 2
# SOLUTIONS TO QUIZ PROBLEM 1

Suppose $x_1, x_2, \ldots, x_n$ is a sample. Then

$$\begin{aligned}
\sum_{i=1}^{n} (x_i - \overline{x})^3 &= \sum_{i=1}^{n} \left[ x_i^3 + 3x_i (\overline{x})^2 - 3x_i^2 \overline{x} - (\overline{x})^3 \right] \\
&= \sum_{i=1}^{n} x_i^3 + 3 \sum_{i=1}^{n} x_i (\overline{x})^2 - 3 \sum_{i=1}^{n} x_i^2 \overline{x} - \sum_{i=1}^{n} (\overline{x})^3 \\
&= \sum_{i=1}^{n} x_i^3 + 3 \sum_{i=1}^{n} x_i (\overline{x})^2 - 3 \sum_{i=1}^{n} x_i^2 \overline{x} - \sum_{i=1}^{n} (\overline{x})^3 \\
&= \sum_{i=1}^{n} x_i^3 + 3 (\overline{x})^2 \sum_{i=1}^{n} x_i - 3\overline{x} \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} (\overline{x})^3 \\
&= \sum_{i=1}^{n} x_i^3 + 3 (\overline{x})^2 n\overline{x} - 3\overline{x} \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} (\overline{x})^3 \\
&= \sum_{i=1}^{n} x_i^3 - 3\overline{x} \sum_{i=1}^{n} x_i^2 + 2n (\overline{x})^3 ,
\end{aligned}$$

where $\overline{x} = (x_1 + x_2 + \cdots + x_n)/n$ denotes the sample mean.