MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 10

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $\mathrm{LN}(0, \theta^2)$. Then $\log X_1, \log X_2, \ldots, \log X_n$ is a random sample from $\mathrm{N}(0, \theta^2)$. So,

$$
\begin{aligned}
& \sum_{i=1}^{n} \left(\frac{\log X_i}{\theta} \right)^2 \sim \chi^2(n) \\
\leftrightarrow \quad & \frac{1}{\theta^2} \sum_{i=1}^{n} (\log X_i)^2 \sim \chi^2(n) \\
\leftrightarrow \quad & \Pr\left[\chi^2_{\frac{\alpha}{2}} < \frac{1}{\theta^2} \sum_{i=1}^{n} (\log X_i)^2 < \chi^2_{1-\frac{\alpha}{2}} \right] = 1 - \alpha \\
\leftrightarrow \quad & \Pr\left[\frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{1-\frac{\alpha}{2}}} < \theta^2 < \frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{\frac{\alpha}{2}}} \right] = 1 - \alpha \\
\leftrightarrow \quad & \Pr\left[\sqrt{\frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{1-\frac{\alpha}{2}}}} < \theta < \sqrt{\frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{\frac{\alpha}{2}}}} \right] = 1 - \alpha.
\end{aligned}
$$

Hence, a $100(1-\alpha)\%$ confidence interval for θ is

$$
\left[\sqrt{\frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{1-\frac{\alpha}{2}}}}, \sqrt{\frac{\sum_{i=1}^{n} (\log X_i)^2}{\chi^2_{\frac{\alpha}{2}}}} \right].
$$