MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 4

Suppose $x_1, x_2, \ldots, x_n$ is a data set from a population with mean μ. Let $\overline{x} = (x_1 + x_2 + \cdots + x_n)/n$ denote the sample mean. Show that

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\begin{aligned}
\sum_{i=1}^{n} (x_i - \mu)^3 &= \sum_{i=1}^{n} \left[(x_i - \overline{x}) + (\overline{x} - \mu) \right]^3 \\
&= \sum_{i=1}^{n} \left[(x_i - \overline{x})^3 + 3 (x_i - \overline{x})^2 (\overline{x} - \mu) + 3 (x_i - \overline{x}) (\overline{x} - \mu)^2 + (\overline{x} - \mu)^3 \right] \\
&= \sum_{i=1}^{n} (x_i - \overline{x})^3 + 3 \sum_{i=1}^{n} (x_i - \overline{x})^2 (\overline{x} - \mu) + \sum_{i=1}^{n} 3 (x_i - \overline{x}) (\overline{x} - \mu)^2 + \sum_{i=1}^{n} (\overline{x} - \mu)^3 \\
&= \sum_{i=1}^{n} (x_i - \overline{x})^3 + 3 (\overline{x} - \mu) \sum_{i=1}^{n} (x_i - \overline{x})^2 + 3 (\overline{x} - \mu)^2 \sum_{i=1}^{n} (x_i - \overline{x}) + n (\overline{x} - \mu)^3 \\
&= \sum_{i=1}^{n} (x_i - \overline{x})^3 + 3 (\overline{x} - \mu) \sum_{i=1}^{n} (x_i - \overline{x})^2 + 3 (\overline{x} - \mu)^2 \left[\left(\sum_{i=1}^{n} x_i \right) - \left(\sum_{i=1}^{n} \overline{x} \right) \right] + n (\overline{x} - \mu)^3 \\
&= \sum_{i=1}^{n} (x_i - \overline{x})^3 + 3 (\overline{x} - \mu) \sum_{i=1}^{n} (x_i - \overline{x})^2 + 3 (\overline{x} - \mu)^2 \left[n\overline{x} - n\overline{x} \right] + n (\overline{x} - \mu)^3 \\
&= n (\overline{x} - \mu)^3 + 3 (\overline{x} - \mu) \sum_{i=1}^{n} (x_i - \overline{x})^2 + \sum_{i=1}^{n} (x_i - \overline{x})^3 .
\end{aligned}
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