MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 3

Suppose X is a negative binomial random variable with probability mass function specified by

$$\Pr(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

for $x = r, r+1, \ldots$.

The expectation can be derived as

$$\begin{aligned}
E(X) &= \sum_{x=r}^{\infty} x \binom{x-1}{r-1} p^r (1-p)^{x-r} \\
&= \sum_{x=r}^{\infty} (x-r+r) \binom{x-1}{r-1} p^r (1-p)^{x-r} \\
&= \sum_{x=r}^{\infty} (x-r) \binom{x-1}{r-1} p^r (1-p)^{x-r} + r \sum_{x=r}^{\infty} \binom{x-1}{r-1} p^r (1-p)^{x-r} \\
&= \left[\sum_{x=r+1}^{\infty} (x-r) \binom{x-1}{r-1} p^r (1-p)^{x-r} \right] + r \\
& \quad \text{since } \binom{x-1}{r-1} p^r (1-p)^{x-r} \text{ is a negative binomial pmf with parameters } r \text{ and } p \\
&= \left[\sum_{x=r+1}^{\infty} (x-r) \frac{(x-1)!}{(r-1)!(x-r)!} p^r (1-p)^{x-r} \right] + r \\
&= \left[\sum_{x=r+1}^{\infty} \frac{(x-1)!}{(r-1)!(x-r-1)!} p^r (1-p)^{x-r} \right] + r \\
&= \left[\sum_{x=r+1}^{\infty} \frac{r(x-1)!}{r(r-1)!(x-r-1)!} p^{r+1-1} (1-p)^{x-r-1+1} \right] + r \\
&= \left[\frac{r(1-p)}{p} \sum_{x=r+1}^{\infty} \frac{(x-1)!}{r!(x-r-1)!} p^{r+1} (1-p)^{x-r-1} \right] + r \\
&= \left[\frac{r(1-p)}{p} \sum_{x=r+1}^{\infty} \binom{x-1}{r} p^{r+1} (1-p)^{x-r-1} \right] + r
\end{aligned}$$

$$
\begin{aligned}
&= \left[\frac{r(1-p)}{p}\right] + r \\
&\quad \text{since } \binom{x-1}{r} p^{r+1}(1-p)^{x-r-1} \text{ is a negative binomial pmf with parameters } r+1 \text{ and } p \\
&= \frac{r}{p}.
\end{aligned}
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