MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 9

Suppose $X_1, \ldots, X_n$ is a random sample from a distribution specified by the cumulative distribution function $F(x) = (x/K)^a$ for $a > 0$ and $0 \leq x \leq K$, where a is known. The cumulative distribution function of $T = \max(X_1, \ldots, X_n)$ is

$$\begin{aligned} F_T(t) &= \Pr(T \leq t) \\ &= \Pr\left(\max\left(X_1, \ldots, X_n\right) \leq t\right) \\ &= \Pr\left(X_1 \leq t, \ldots, X_n \leq t\right) \\ &= \Pr\left(X_1 \leq t\right) \cdots \Pr\left(X_n \leq t\right) \\ &= \left[\Pr\left(X \leq t\right)\right]^n \\ &= \left[F\left(t\right)\right]^n \\ &= \left(\frac{t}{K}\right)^{na}. \end{aligned}$$

Setting

$$F_T(t) = \frac{\alpha}{2}$$

and

$$F_T(t) = 1 - \frac{\alpha}{2}$$

gives

$$t = K\left(\frac{\alpha}{2}\right)^{1/(na)}$$

and

$$t = K\left(1 - \frac{\alpha}{2}\right)^{1/(na)}.$$

So,

$$\Pr\left(K\left(\frac{\alpha}{2}\right)^{1/(na)} < T < K\left(1 - \frac{\alpha}{2}\right)^{1/(na)}\right) = 1 - \alpha$$

which is equivalent to

$$\Pr\left(T\left(1 - \frac{\alpha}{2}\right)^{-1/(na)} < K < T\left(\frac{\alpha}{2}\right)^{-1/(na)}\right) = 1 - \alpha.$$

Hence, a $100(1-\alpha)$ percent confidence interval for K is

$$\left[T\left(1-\frac{\alpha}{2}\right)^{-1/(na)}, T\left(\frac{\alpha}{2}\right)^{-1/(na)}\right].$$

So, the correct answer is d).