

**MATH10282: INTRODUCTION TO STATISTICS  
SEMESTER 2  
SOLUTIONS TO QUIZ PROBLEM 4**

Suppose that a random variable  $X$  has the cumulative distribution function

$$F_X(x) = 1 - (1 - x^a)^b$$

for  $0 < x < 1$ ,  $a > 0$  and  $b > 0$ . To determine the quantile function  $Q(p)$ , we solve

$$F_X(Q(p)) = p$$

which is equivalent to

$$1 - (1 - (Q(p))^a)^b = p$$

which is equivalent to

$$(1 - (Q(p))^a)^b = 1 - p$$

which is equivalent to

$$1 - (Q(p))^a = (1 - p)^{1/b}$$

which is equivalent to

$$(Q(p))^a = 1 - (1 - p)^{1/b}$$

which is equivalent to

$$Q(p) = \left[1 - (1 - p)^{1/b}\right]^{1/a}.$$

Hence,

$$Q(p) = \left[1 - (1 - p)^{1/b}\right]^{1/a}.$$

The IQR is

$$\begin{aligned} Q(0.75) - Q(0.25) &= \left[1 - (1 - 0.75)^{1/b}\right]^{1/a} - \left[1 - (1 - 0.25)^{1/b}\right]^{1/a} \\ &= \left[1 - (0.25)^{1/b}\right]^{1/a} - \left[1 - (0.75)^{1/b}\right]^{1/a}. \end{aligned}$$

**So, the correct answer is d).**