MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 3

Suppose that $X_1, \ldots, X_n$ is a random sample from a Poisson distribution with parameter λ. Then

$$\begin{aligned}
\Pr(X_1 = 1, X_2 = 2, \ldots, X_n = n) &= \Pr(X_1 = 1)\Pr(X_2 = 2)\cdots\Pr(X_n = n) \\
&= \prod_{i=1}^{n} \Pr(X_i = i) \\
&= \prod_{i=1}^{n} \frac{\lambda^i \exp(-\lambda)}{i!} \\
&= \left(\prod_{i=1}^{n} \lambda^i\right)\left(\prod_{i=1}^{n} \exp(-\lambda)\right)\left(\prod_{i=1}^{n} \frac{1}{i!}\right) \\
&= \left(\lambda^{1+2+\cdots+n}\right)(\exp(-n\lambda))\left(\prod_{i=1}^{n} \frac{1}{i!}\right) \\
&= \lambda^{\frac{n(n+1)}{2}} \exp(-n\lambda)\left(\prod_{i=1}^{n} \frac{1}{i!}\right).
\end{aligned}$$

So, the correct answer is a).