MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 10

Suppose $X_1, \ldots, X_n$ is a random sample from a distribution specified by the cumulative distribution function $F(x) = (x/K)^a$ for $a > 0$ and $0 \leq x \leq K$, where a is known. Consider testing $H_0 : K = K_0$ versus $H_1 : K > K_0$. Suppose we reject H_0 when $T = \max(X_1, \ldots, X_n) < c$ for some constant $c < K$. The probability of type II error is

$$
\begin{aligned}
\Pr\left(\text{Type II error}\right) &= \Pr\left(T \geq c \mid K > K_0\right) \\
&= 1 - \Pr\left(T < c \mid K > K_0\right) \\
&= 1 - \Pr\left(\max(X_1, \ldots, X_n) < c\right) \\
&= 1 - \Pr\left(X_1 < c, \ldots, X_n < c\right) \\
&= 1 - \Pr\left(X_1 < c\right) \cdots \Pr\left(X_n < c\right) \\
&= 1 - \left[\Pr\left(X < c\right)\right]^n \\
&= 1 - \left[1 - F(c)\right]^n \\
&= 1 - \left(\frac{c}{K}\right)^{na}.
\end{aligned}
$$

We must have

$$
1 - \left(\frac{c}{K}\right)^{na} \leq \beta
$$

which is equivalent to

$$
\left(\frac{c}{K}\right)^{na} \geq 1 - \beta
$$

which is equivalent to

$$
na \log\left(\frac{c}{K}\right) \geq \log(1 - \beta)
$$

which is equivalent to

$$
n \leq \frac{\log(1 - \beta)}{a \log\left(\frac{c}{K}\right)}.
$$

So, the correct answer is c).