MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 9

Suppose $X_1, \ldots, X_n$ is a random sample from a distribution specified by the cumulative distribution function $F(x) = 1 - (K/x)^a$ for $a > 0$ and $x \geq K > 0$, where a is known. The cumulative distribution function of $T = \min(X_1, \ldots, X_n)$ is

$$
\begin{aligned}
F_T(t) &= \Pr(T \leq t) \\
&= 1 - \Pr(T > t) \\
&= 1 - \Pr\left(\min(X_1, \ldots, X_n) > t\right) \\
&= 1 - \Pr\left(X_1 > t, \ldots, X_n > t\right) \\
&= 1 - \Pr\left(X_1 > t\right) \cdots \Pr\left(X_n > t\right) \\
&= 1 - \left[\Pr\left(X > t\right)\right]^n \\
&= 1 - \left[1 - \Pr\left(X \leq t\right)\right]^n \\
&= 1 - \left[1 - F\left(t\right)\right]^n \\
&= 1 - \left(\frac{K}{t}\right)^{na}.
\end{aligned}
$$

Setting

$$F_T(t) = \frac{\alpha}{2}$$

and

$$F_T(t) = 1 - \frac{\alpha}{2}$$

gives

$$t = K\left(1 - \frac{\alpha}{2}\right)^{-1/(na)}$$

and

$$t = K\left(\frac{\alpha}{2}\right)^{-1/(na)}.$$

So,

$$\Pr\left(K\left(1 - \frac{\alpha}{2}\right)^{-1/(na)} < T < K\left(\frac{\alpha}{2}\right)^{-1/(na)}\right) = 1 - \alpha$$

which is equivalent to

$$\Pr\left(T\left(\frac{\alpha}{2}\right)^{1/(na)} < K < T\left(1-\frac{\alpha}{2}\right)^{1/(na)}\right) = 1-\alpha.$$

Hence, a $100(1-\alpha)$ percent confidence interval for K is

$$\left[T\left(\frac{\alpha}{2}\right)^{1/(na)}, T\left(1-\frac{\alpha}{2}\right)^{1/(na)}\right].$$

So, the correct answer is a).