MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 7

Suppose $X_i \sim Exp\,(i\lambda)$, $i = 1, \dots, n$ are independent random variables. The likelihood function of λ is

$$L(\lambda) = \prod_{i=1}^{n} i\lambda \exp\left(-i\lambda x_i\right) = n!\lambda^n \exp\left(-\lambda \sum_{i=1}^{n} i x_i\right).$$

The log-likelihood function is

$$\log L(\lambda) = \log n! + n \log \lambda - \lambda \sum_{i=1}^{n} i x_i.$$

The derivative with respect to λ is

$$\frac{d \log L(\lambda)}{d\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} i x_i.$$

Setting this to zero and solving for λ, we obtain

$$\widehat{\lambda} = \frac{n}{\sum_{i=1}^{n} i x_i}.$$

This is a maximum likelihood estimator since

$$\frac{d^2 \log L(\lambda)}{d\lambda^2} = -\frac{n}{\lambda^2} < 0.$$

So, the correct answer is c).