MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 4

Suppose X has the cumulative distribution function

$$F_X(x) = \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$. To determine the quantile function $Q(p)$, we solve

$$F_X\left(Q(p)\right) = p$$

which is equivalent to

$$\exp\left[-\exp\left(-\frac{Q(p)-\mu}{\sigma}\right)\right] = p$$

which is equivalent to

$$\exp\left(-\frac{Q(p)-\mu}{\sigma}\right) = -\log p$$

which is equivalent to

$$\frac{Q(p)-\mu}{\sigma} = -\log\left(-\log p\right).$$

Hence,

$$Q(p) = \mu - \sigma\log\left(-\log p\right).$$

The IQR is

$$\begin{aligned} Q(0.75) - Q(0.25) &= \left[\mu - \sigma\log\left(-\log 0.75\right)\right] - \left[\mu - \sigma\log\left(-\log 0.25\right)\right] \\ &= \sigma\left[\log\left(-\log 0.25\right) - \log\left(-\log 0.75\right)\right]. \end{aligned}$$

So, the correct answer is d).