MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 2

Suppose that $X_1, \ldots, X_n$ is a random sample from a distribution specified by the probability density function

$$f_X(x) = ax^{a-1}$$

for $a > 0$ and $0 < x < 1$. From the lecture notes, the mean and variance of the sampling distribution of $\overline{X}$ are

$$E\left(\overline{X}\right) = \mu$$

and

$$Var\left(\overline{X}\right) = \frac{\sigma^2}{n},$$

where $\mu = E(X)$ and $\sigma^2 = Var(X)$. From the given probability density function,

$$
\begin{aligned}
\mu &= \int_0^1 xax^{a-1}dx \\
&= a\left[\frac{x^{a+1}}{a+1}\right]_0^1 \\
&= a\left[\frac{1}{a+1} - 0\right] \\
&= \frac{a}{a+1}
\end{aligned}
$$

and

$$
\begin{aligned}
\sigma^2 &= E\left(X^2\right) - [E(X)]^2 \\
&= \int_0^1 x^2 ax^{a-1}dx - \left(\frac{a}{a+1}\right)^2 \\
&= a\left[\frac{x^{a+2}}{a+2}\right]_0^1 - \left(\frac{a}{a+1}\right)^2 \\
&= a\left[\frac{1}{a+2} - 0\right] - \left(\frac{a}{a+1}\right)^2 \\
&= \frac{a}{a+2} - \left(\frac{a}{a+1}\right)^2 \\
&= \frac{a}{(a+2)(a+1)^2}.
\end{aligned}
$$

Hence,

$$E\left(\overline{X}\right) = \frac{a}{a+1}$$

and

$$Var\left(\overline{X}\right) = \frac{a}{n(a+2)(a+1)^2}.$$

So, the correct answer is c).