MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 10

Suppose $X_1, \ldots, X_n$ is a random sample from a distribution specified by the cumulative distribution function $F(x) = 1 - (K/x)^a$ for $a > 0$ and $x \geq K > 0$, where a is known. Consider testing $H_0 : K = K_0$ versus $H_1 : K > K_0$. Suppose we reject H_0 when $T = \min(X_1, \ldots, X_n) > c$ for some constant c. The probability of type II error is

$$\begin{aligned}
\Pr(\text{Type II error}) &= \Pr(T \leq c \mid K > K_0) \\
&= 1 - \Pr(T > c \mid K > K_0) \\
&= 1 - \Pr(\min(X_1, \ldots, X_n) > c) \\
&= 1 - \Pr(X_1 > t, \ldots, X_n > C) \\
&= 1 - \Pr(X_1 > c) \cdots \Pr(X_n > c) \\
&= 1 - [\Pr(X > c)]^n \\
&= 1 - [1 - \Pr(X \leq c)]^n \\
&= 1 - [1 - F(c)]^n \\
&= 1 - \left(\frac{K}{c}\right)^{na}.
\end{aligned}$$

We must have

$$1 - \left(\frac{K}{c}\right)^{na} \leq \beta$$

which is equivalent to

$$\left(\frac{K}{c}\right)^{na} \geq 1 - \beta$$

which is equivalent to

$$na \log\left(\frac{K}{c}\right) \geq \log(1 - \beta)$$

which is equivalent to

$$n \geq \frac{\log(1-\beta)}{a \log\left(\frac{K}{c}\right)}.$$

So, the correct answer is d).