

MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 1

Suppose you have the following data: $1, 2, c, d, 6$ where c and d are unknown. Since the sample mean is 2, we have

$$\frac{1 + 2 + c + d + 6}{5} = 2$$

which implies

$$\frac{c + d + 9}{5} = 2$$

which implies

$$c + d = 1. \tag{1}$$

I use the following formula for sample variance

$$s^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right).$$

Since the sample variance is 6, we have

$$\frac{1}{4} (1 + 4 + c^2 + d^2 + 36 - 5 \cdot 4) = 6$$

which implies

$$c^2 + d^2 + 21 = 24$$

which implies

$$c^2 + d^2 = 3. \tag{2}$$

From (1), $d = 1 - c$. Substituting into (2), we obtain $c^2 + (1 - c)^2 = 3$, which is equivalent to $c^2 - c - 1 = 0$. The roots of this equation are $c = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$. The corresponding values for d are $d = \frac{1}{2} \mp \frac{\sqrt{5}}{2}$.

So, the correct answer is a).