

MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 9

The proportion p of defective items in a large population is unknown. We wish to test $H_0 : p = 0.2$ versus $H_1 : p \neq 0.2$. Suppose a random sample of 20 items is drawn from the population. Let Y be the number of defective items in the sample. Consider the test which rejects H_0 if and only if $Y \geq 7$ or $Y \leq 1$.

Clearly, Y has a binomial distribution with parameters 20 and p . So,

$$\begin{aligned} P(\text{Type II Error}) &= P(\text{Accept } H_0 | p \neq 0.2) \\ &= P(2 \leq Y \leq 6 | p \neq 0.2) \\ &= \sum_{i=2}^6 P(Y = i | p \neq 0.2) \\ &= \sum_{i=2}^6 \binom{20}{i} p^i (1-p)^{20-i}. \end{aligned}$$