MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 4

Let $x_1, x_2, \ldots, x_n$ denote a data set and let $\overline{x} = (x_1 + \cdots + x_n)/n$. Then

$$\begin{aligned}
\sum_{i=1}^{n} (x_i - \overline{x})^4 &= \sum_{i=1}^{n} \left[x_i^4 - 4x_i^3\overline{x} + 6x_i^2 (\overline{x})^2 - 4x_i (\overline{x})^3 + (\overline{x})^4 \right] \\
&= \sum_{i=1}^{n} x_i^4 - 4\sum_{i=1}^{n} x_i^3\overline{x} + 6\sum_{i=1}^{n} x_i^2 (\overline{x})^2 - 4\sum_{i=1}^{n} x_i (\overline{x})^3 + \sum_{i=1}^{n} (\overline{x})^4 \\
&= \sum_{i=1}^{n} x_i^4 - 4\overline{x}\sum_{i=1}^{n} x_i^3 + 6(\overline{x})^2 \sum_{i=1}^{n} x_i^2 - 4(\overline{x})^3 \sum_{i=1}^{n} x_i + n(\overline{x})^4 \\
&= \sum_{i=1}^{n} x_i^4 - 4\overline{x}\sum_{i=1}^{n} x_i^3 + 6(\overline{x})^2 \sum_{i=1}^{n} x_i^2 - 4(\overline{x})^3 (n\overline{x}) + n(\overline{x})^4 \\
&= \sum_{i=1}^{n} x_i^4 - 4\overline{x}\sum_{i=1}^{n} x_i^3 + 6(\overline{x})^2 \sum_{i=1}^{n} x_i^2 - 4n(\overline{x})^4 + n(\overline{x})^4 \\
&= \sum_{i=1}^{n} x_i^4 - 4\overline{x}\sum_{i=1}^{n} x_i^3 + 6(\overline{x})^2 \sum_{i=1}^{n} x_i^2 - 3n(\overline{x})^4 .
\end{aligned}$$