MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
SOLUTIONS TO QUIZ PROBLEM 2

Let $x_1, x_2, \ldots, x_n$ denote a data set from a population with mean μ. Let $\overline{x} = \left(x_1 + \cdots + x_n\right)/n$. Then

$$\begin{aligned}
\sum_{i=1}^{n} \left(x_i - \mu\right)^4 &= \sum_{i=1}^{n} \left[\left(x_i - \overline{x}\right) + \left(\overline{x} - \mu\right)\right]^4 \\
&= \sum_{i=1}^{n} \left[\left(x_i - \overline{x}\right)^4 + 4\left(x_i - \overline{x}\right)^3 \left(\overline{x} - \mu\right) + 6\left(x_i - \overline{x}\right)^2 \left(\overline{x} - \mu\right)^2 + 4\left(x_i - \overline{x}\right)\left(\overline{x} - \mu\right)^3 + \left(\overline{x} - \mu\right)^4\right] \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 \left(\overline{x} - \mu\right) + 6\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \left(\overline{x} - \mu\right)^2 \\
&\quad 4\sum_{i=1}^{n} \left(x_i - \overline{x}\right)\left(\overline{x} - \mu\right)^3 + \sum_{i=1}^{n} \left(\overline{x} - \mu\right)^4 \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\left(\overline{x} - \mu\right)\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 + 6\left(\overline{x} - \mu\right)^2 \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \\
&\quad +4\left(\overline{x} - \mu\right)^3 \sum_{i=1}^{n} \left(x_i - \overline{x}\right) + n\left(\overline{x} - \mu\right)^4 \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\left(\overline{x} - \mu\right)\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 + 6\left(\overline{x} - \mu\right)^2 \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \\
&\quad +4\left(\overline{x} - \mu\right)^3 \sum_{i=1}^{n} \left[\left(\sum_{i=1}^{n} x_i\right) - \left(\sum_{i=1}^{n} \overline{x}\right)\right] + n\left(\overline{x} - \mu\right)^4 \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\left(\overline{x} - \mu\right)\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 + 6\left(\overline{x} - \mu\right)^2 \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \\
&\quad +4\left(\overline{x} - \mu\right)^3 \sum_{i=1}^{n} \left[\left(\sum_{i=1}^{n} x_i\right) - \left(n\overline{x}\right)\right] + n\left(\overline{x} - \mu\right)^4 \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\left(\overline{x} - \mu\right)\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 + 6\left(\overline{x} - \mu\right)^2 \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \\
&\quad +4\left(\overline{x} - \mu\right)^3 \sum_{i=1}^{n} \left[\left(\sum_{i=1}^{n} x_i\right) - \left(\sum_{i=1}^{n} x_i\right)\right] + n\left(\overline{x} - \mu\right)^4 \\
&= \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4 + 4\left(\overline{x} - \mu\right)\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^3 + 6\left(\overline{x} - \mu\right)^2 \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 + n\left(\overline{x} - \mu\right)^4.
\end{aligned}$$