

**MATH10282: INTRODUCTION TO STATISTICS  
SEMESTER 2  
SOLUTIONS TO QUIZ PROBLEM 2**

Let  $x_1, x_2, \dots, x_n$  denote a data set and let  $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$  denote the order statistics in ascending order.

Note that  $r = p(n + 1)$  and  $r' = [p(n + 1)]$  are

$$r = \begin{cases} m + \frac{1}{4}, & \text{if } n = 4m, \\ m, & \text{if } n = 4m - 1, \\ m - \frac{1}{4}, & \text{if } n = 4m - 2, \\ m - \frac{1}{2}, & \text{if } n = 4m - 3 \end{cases}$$

and

$$r' = \begin{cases} m, & \text{if } n = 4m, \\ m, & \text{if } n = 4m - 1, \\ m - 1, & \text{if } n = 4m - 2, \\ m - 1, & \text{if } n = 4m - 3, \end{cases}$$

respectively. So,

$$r - r' = \begin{cases} \frac{1}{4}, & \text{if } n = 4m, \\ 0, & \text{if } n = 4m - 1, \\ \frac{3}{4}, & \text{if } n = 4m - 2, \\ \frac{1}{2}, & \text{if } n = 4m - 3. \end{cases}$$

Hence,

$$\text{first quartile} = \begin{cases} x_{(m)} + \frac{1}{4} [x_{(m+1)} - x_{(m)}], & \text{if } n = 4m, \\ x_{(m)}, & \text{if } n = 4m - 1, \\ x_{(m-1)} + \frac{3}{4} [x_{(m)} - x_{(m-1)}], & \text{if } n = 4m - 2, \\ x_{(m-1)} + \frac{1}{2} [x_{(m)} - x_{(m-1)}], & \text{if } n = 4m - 3. \end{cases}$$