

MATH10282: INTRODUCTION TO STATISTICS
SEMESTER 2
QUIZ PROBLEM 10
(Deadline: Tuesday 5 May 2020, 9:00am)

Suppose X_1, X_2, \dots, X_n is an independent random sample from $\text{Exp}(\theta)$. Consider the test for $H_0 : \theta = \theta_1$ versus $H_1 : \theta = \theta_2$ with

$$\sum_{i=1}^n X_i < c$$

as the rule for rejecting H_0 . Show that the probability of type II error is

$$\left[1 + \theta_2 c + \frac{\theta_2^2 c^2}{2} + \dots + \frac{\theta_2^{n-1} c^{n-1}}{(n-1)!} \right] \exp(-\theta_2 c).$$

You may use mathematical induction to prove this.

This problem is worth 1 mark. Marking scheme: 1 mark if the answer is correct, and the derivation is correct and detailed enough; 0.5 mark if the answer is correct, and the derivation is incorrect or not detailed enough; 0.5 mark if the answer is incorrect or not given, but the derivation is correct and detailed enough; 0 mark if the answer is correct, but the derivation is not detailed enough; 0 mark if the answer is incorrect, and the derivation is not detailed enough.

Please email your submission directly to me, mbbssn2@manchester.ac.uk I will mark your solutions and email your mark and your scanned working to you within 24 hours of the deadline. **PLEASE DO NOT FORGET TO WRITE YOUR FULL NAME AND ID.**