LECTURE

9 NOVEMBER

10:00-11:00AM

MATH4/68181

Estimation methods for VaR

a) Parametric estimation methods

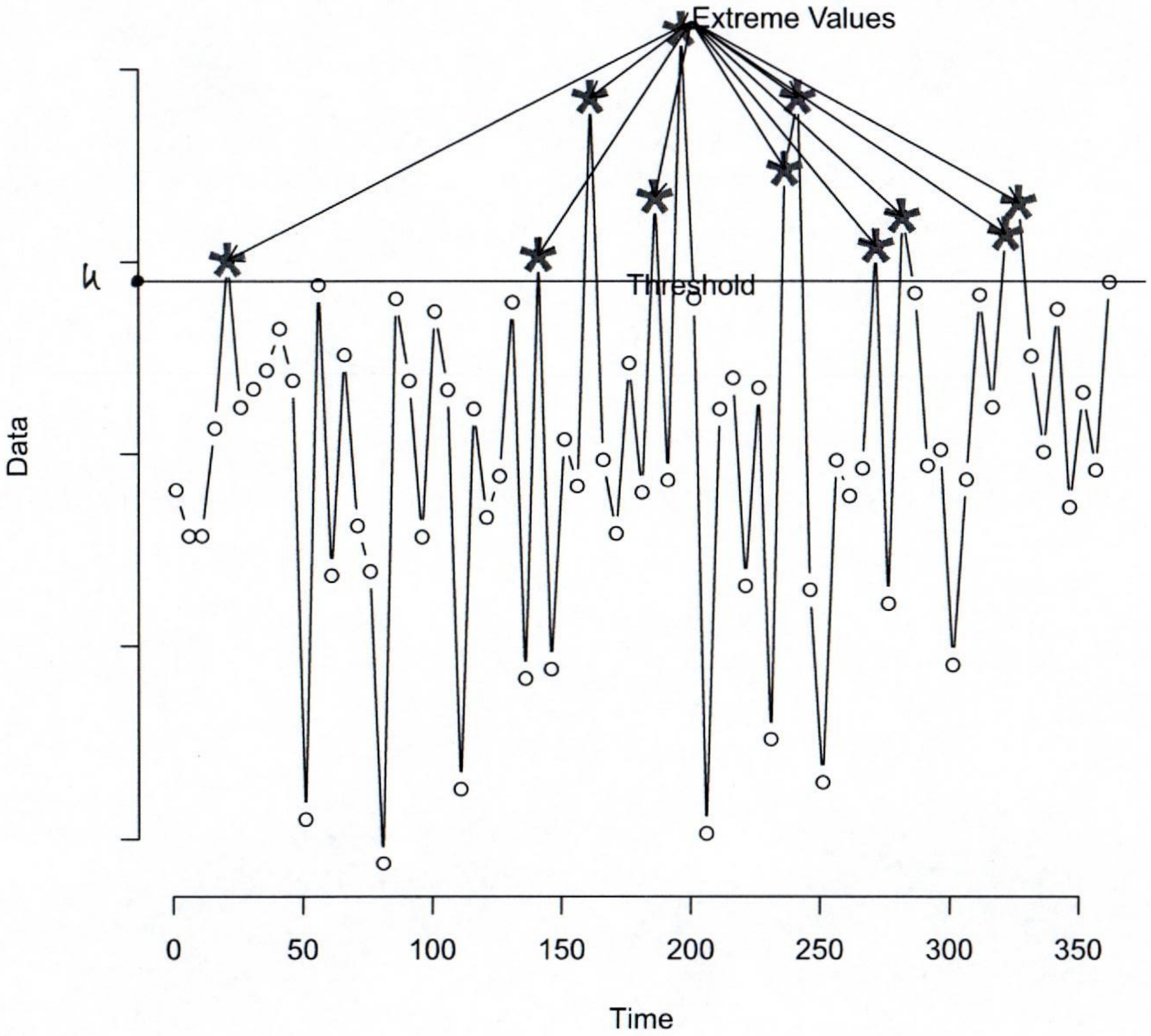
b) Non-parametric " "

c) Semi-parametric " "

a) b) Level 3

a) b) c) Levels 4, 6

Definition 2



c) Semi-parametric estimation methods

1) Extreme value method

Suppose Definition I of extreme value.

Let $M_n = \max(X_1, X_2, \ldots, X_n)$.

By the ETT, we can write

$$P(M_n < x) = e^{-\left(1+\xi \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}}$$

where $-\infty < \xi < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$.

To find VaR, set

$$P(M_n < x) = p$$

$$\Rightarrow e^{-\left(1+\xi \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}} = p$$

$$\Rightarrow -\left(1+\xi \frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} = \log p$$

$$\Rightarrow 1+\xi \frac{x-\mu}{\sigma} = (-\log p)^{-\xi}$$

$$\Rightarrow x = \mu + \frac{\sigma}{\xi}\left[(-\log p)^{-\xi} - 1\right]$$

$$\Rightarrow VaR_p = \mu + \frac{\sigma}{\xi}\left[(-\log p)^{-\xi} - 1\right]$$

How to estimate VaR_p?

Estimation of ξ

Suppose $X_1, X_2, \ldots$ are IID observations. Then two possible estimators of ξ are

$$\hat{\xi} = \frac{1}{k} \sum_{i=1}^{k} \log \frac{X_{(i)}}{X_{(i+1)}} \quad \cdots \quad (*)$$

and

$$\hat{\xi} = \frac{1}{\log 2} \log \frac{X_{(k+1)} - X_{(2k+1)}}{X_{(2k+1)} - X_{(4k+1)}} \quad \cdots (**)$$

where $k \geq 1$ is an integer and

$X_{(1)} \geq X_{(2)} \geq \cdots$ are deseending order statistics

(*) is due to Hill (1975)

(* *) is due to Pickands (1975)

(*) and (**) involve just the order statistics. They do not suppose any distribution for the data. Hence, (*) and (**) are non-parametric.

Estimation of μ and σ

Let L denote the likelihood for $X_1, X_2, \ldots, X_n$ following the GEV distribution. Then the MLEs of μ and σ say $\hat{\mu}$ and $\hat{\sigma}$ are the simultaneous solutions of

$$\frac{\partial \log L}{\partial \mu} = 0$$

and

$$\frac{\partial \log L}{\partial \sigma} = 0 .$$

Hence, the estimator of VaR_p is

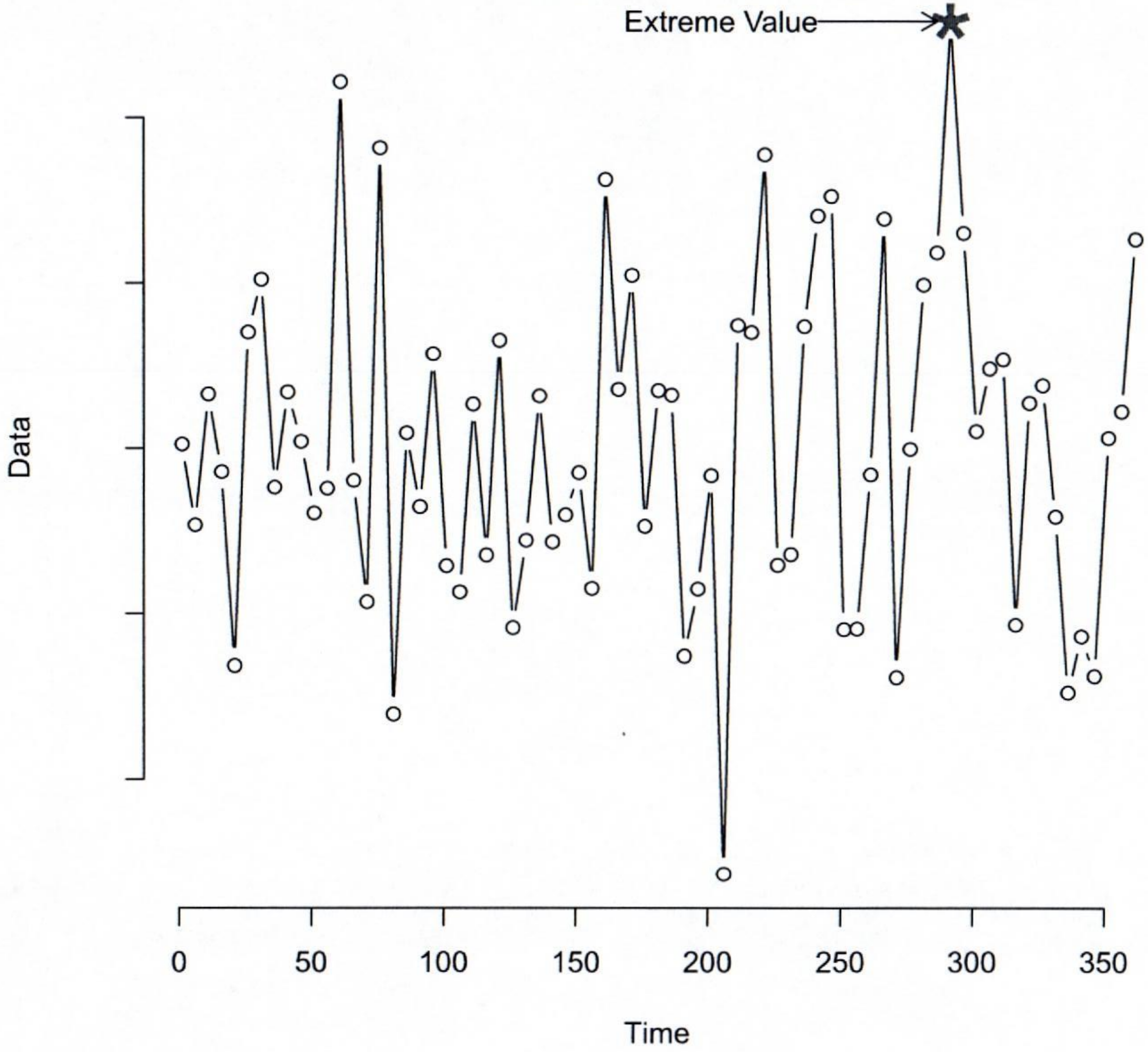
$$\widehat{VaR_p} = \hat{\mu} + \frac{\hat{\sigma}}{\hat{\xi}}\left[(-\log p)^{-\hat{\xi}} - 1\right]$$

Parametric estimators

Non-parametric estimator

Hence, $\widehat{VaR_p}$ is a semi-parametric estimator.

Definition 1



2. Generalized Pareto method

Suppose definition 2 of extreme values.

Let X = variable of interest

u = threshold

F = CDF of X.

From what we covered before,

$$F(x) = 1 - q\left(1 + \frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}}$$

where $q = P(X > u)$, $\sigma > 0$ and $-\infty < \xi < \infty$.

To find VaR, set

$$F(x) = p$$

$$\Rightarrow 1 - q\left(1 + \frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}} = p$$

$$\Rightarrow \left(1 + \frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}} = \frac{1-p}{q}$$

$$\Rightarrow 1 + \frac{x-u}{\sigma} = \left(\frac{1-p}{q}\right)^{-\xi}$$

$$\Rightarrow x = u + \sigma\left[\left(\frac{1-p}{q}\right)^{-\xi} - 1\right].$$

Hence,

$$VaR_p(X) = u + \sigma\left[\left(\frac{1-p}{q}\right)^{-\xi} - 1\right].$$

Take $\hat{\xi}$ ~~are~~ as the estimators due to Hill (1975) and Pickands (1975). These are non-parametric estimators.

Take $\hat{\sigma}$ as the MLE of σ. This is a parametric estimator.

Estimate q by

$$\hat{q} = \frac{1}{n} \sum_{i=1}^{n} I\{X_i > u\}$$

(proportion of data exceeding u).

Hence, the estimator of VaR_p is

$$\widehat{VaR_p}(X) = u + \hat{\sigma}\left[\left(\frac{1-p}{\hat{q}}\right)^{-\hat{\xi}} - 1\right].$$

Clearly, this is a semi-parametric estimator.